On the shape of the mass-function of dense cores in the Hi-GAL fields: frequentist vs. Bayesian approach

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TALK OUTLINE

- IMF & CMF
- Hi-GAL data
- Bayesian vs. frequentist approach
- Standard Frequentist approach
- Bayesian inference

IMF & CMF

- A theory of SF must explain the origin of the stellar **IMF**. This involves the **whole SF process**

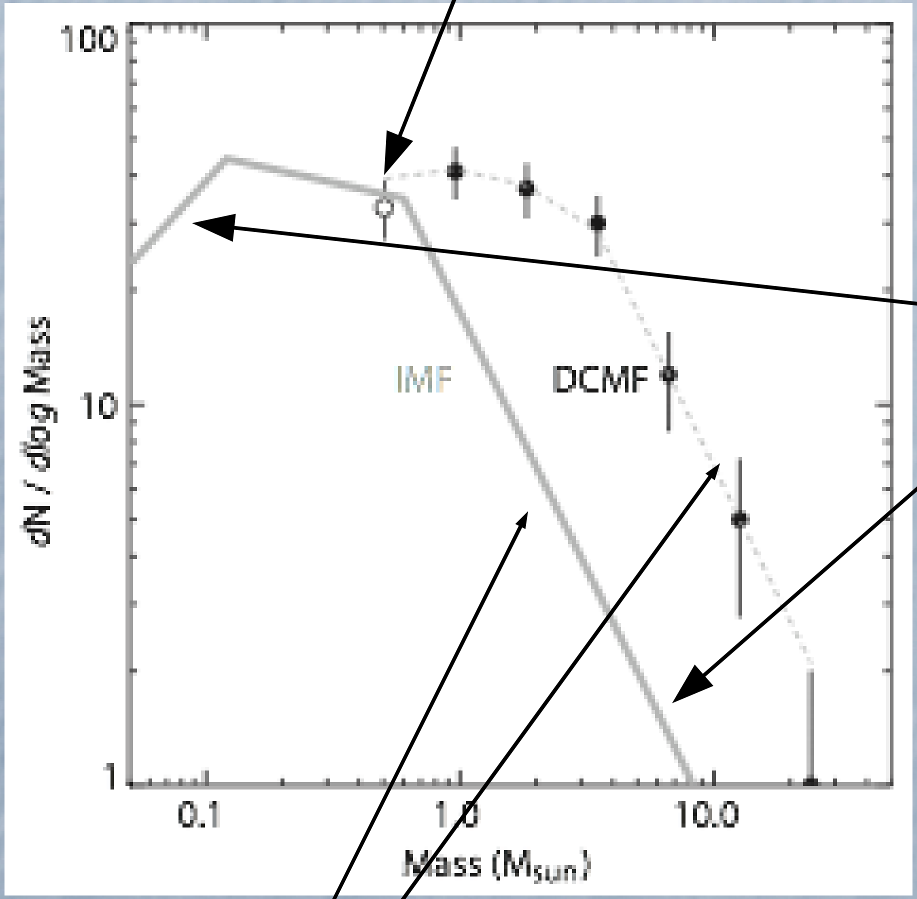
- Stars form from dense cores of molecular gas and dust ⇒ relationship between CMF and IMF contains information regarding **how cores evolve into stars**

- CMFs are often different but **IMF is “universal”**.

(a) different mechanism(s) (in different environments) always produce the same IMF. **Or,**
(b) there is a single, underlying, mechanism that produces the same IMF in all environments.

MAIN PROPERTIES OF IMF

characteristic mass <~ 1M⊙



Universal, with deviations and mass cut-off, at both ends

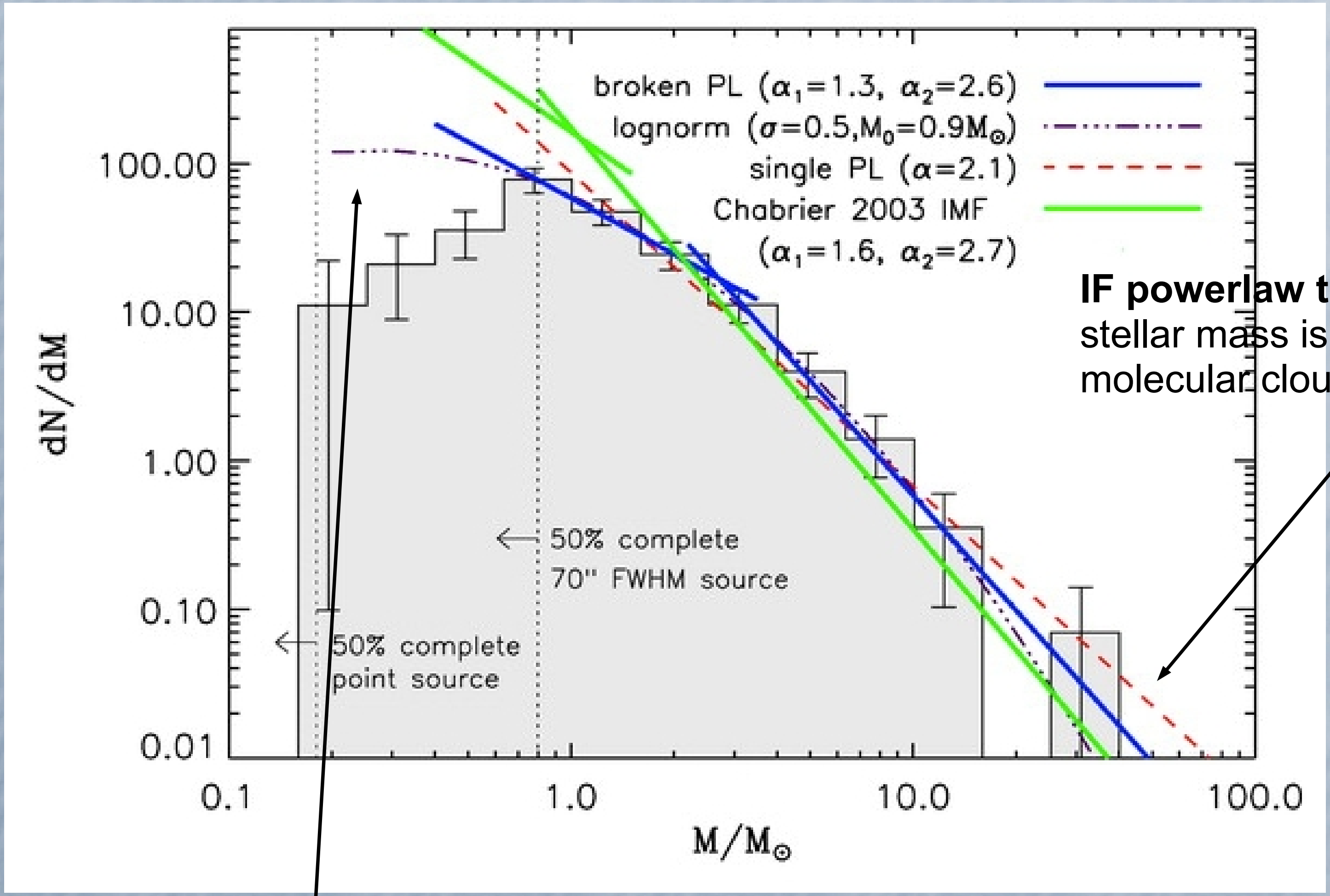
(Alves et al. 2007)

Similarity between IMF and CMF

⇒ **IMF is set early in the SF process**, but how early?

WHAT CAN WE LEARN FROM THE CMF?

To understand how cores produce the full spectrum of stellar masses, it is essential to understand the **probability distribution function (PDF)** from which the CMF is drawn.



IF powerlaw tail extends to HM then stellar mass is determined during the molecular cloud phase

(Enoch et al. 2006)

A **lognormal** CMF would disfavor the idea that **massive stars** form directly from massive cores , and may imply that massive stars form through mechanisms distinct from LM stars

OBSERVATIONAL PROBLEMS

- **“A given CMF evolved according to different evolutionary pathways produces <u>variations in the resultant IMF that are insignificant</u> in relation to the errors inherent in current samples of dense cores.”** (Swift & Williams 2010)

- **Distinguishing between the various forms of CMF** is complicated:
  (a) must measure CMF over large dynamic ranges
  (b) lognormal and powerlaw forms can look quite similar over limited mass ranges.

- The **Hi-GAL survey** provides 1000s of new cores, but still some issues: distance estimates, angular resolution and area-averaging

Main Approaches to Statistics

➢ **Frequentists**:

- **Probability is objective and refers to the limit of an event's relative frequency in a large number of trials.**
- Parameters are all fixed and unknown constants.
- Any statistical process only has interpretations based on limited frequencies. For example, a 95% C.I. of a given parameter will contain the true value of the parameter 95% of the time.

➢ **Bayesians**:

- **Probability is subjective and can be applied to single events based on degree of confidence or beliefs.**
- Parameters are random variables that has a given distribution, and other probability statements can be made about them.
- Probability has a distribution over the parameters, and point estimates are usually done by either taking the mode or the mean of the distribution.

Bayesian Statistics

- Bayesian approach to statistical inference is based on axiomatic foundations, providing a unifying logical structure

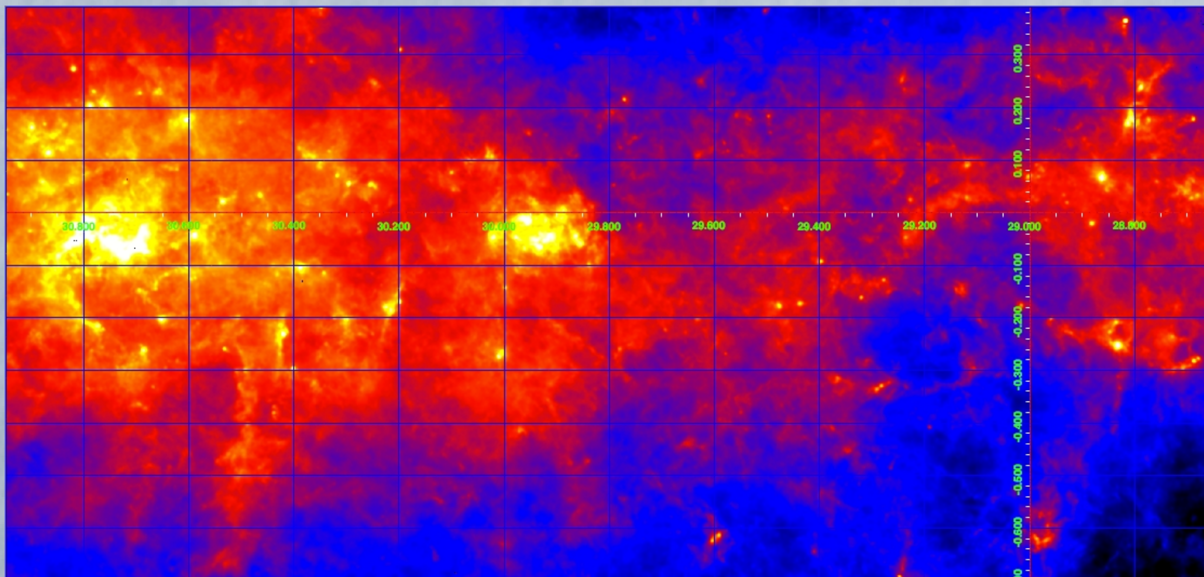
- Bayesian methods may be applied to highly structured complex problems, often untractable by traditional statistical methods.

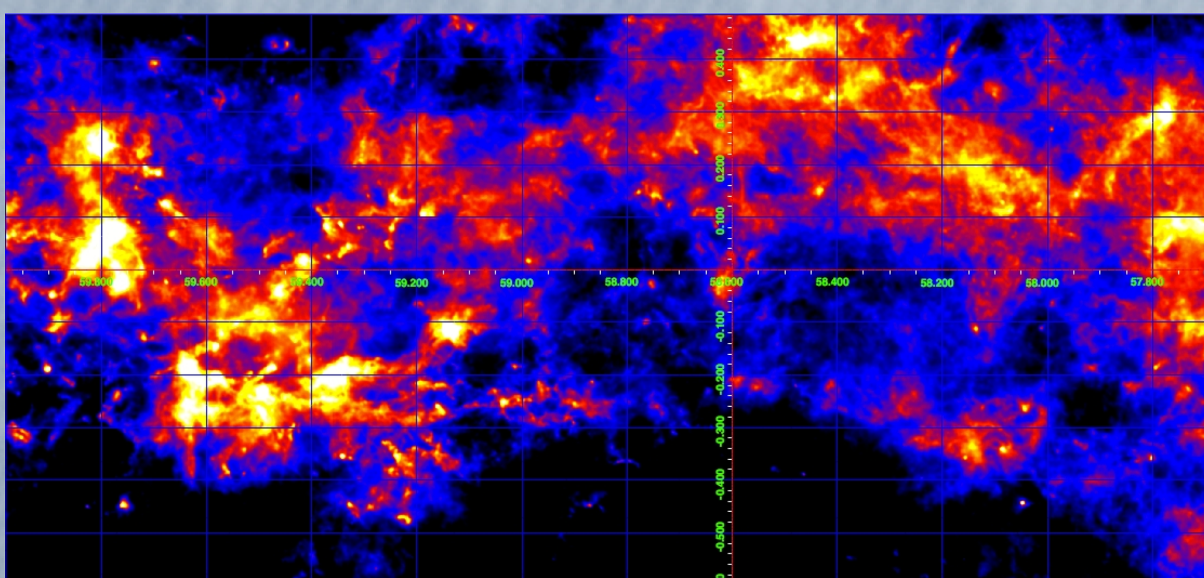
- Parameters are treated as random variables. Not a description of their variability (parameters are typically *fixed, unknown* quantities) but a description of the *uncertainty* about their true values.

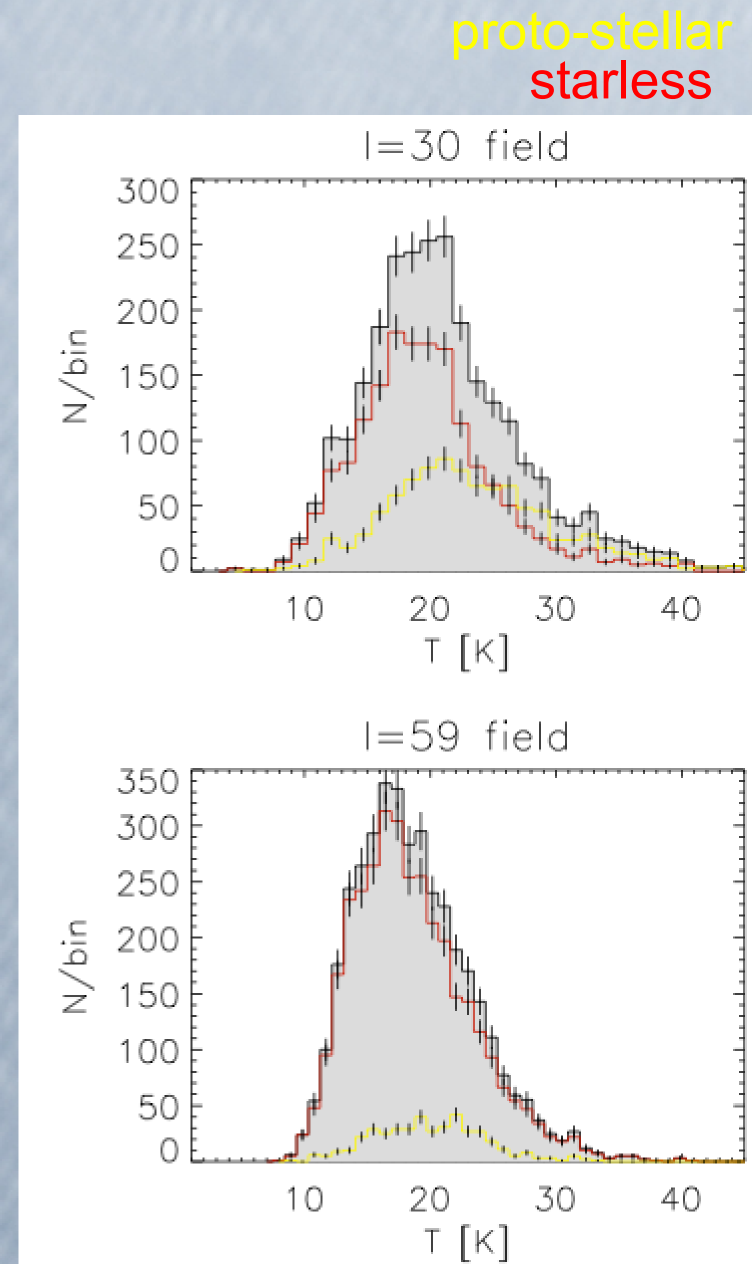
Occurrences of “Bayes” in 'abstract' of ADS

	PHYSICS	ASTRONOMY	**ALL**
1950 - 1999	1612	372	**1901**
2000 - 2012	5709 (↑ 3.5x)	1672 (↑ 4.5x)	**6953**

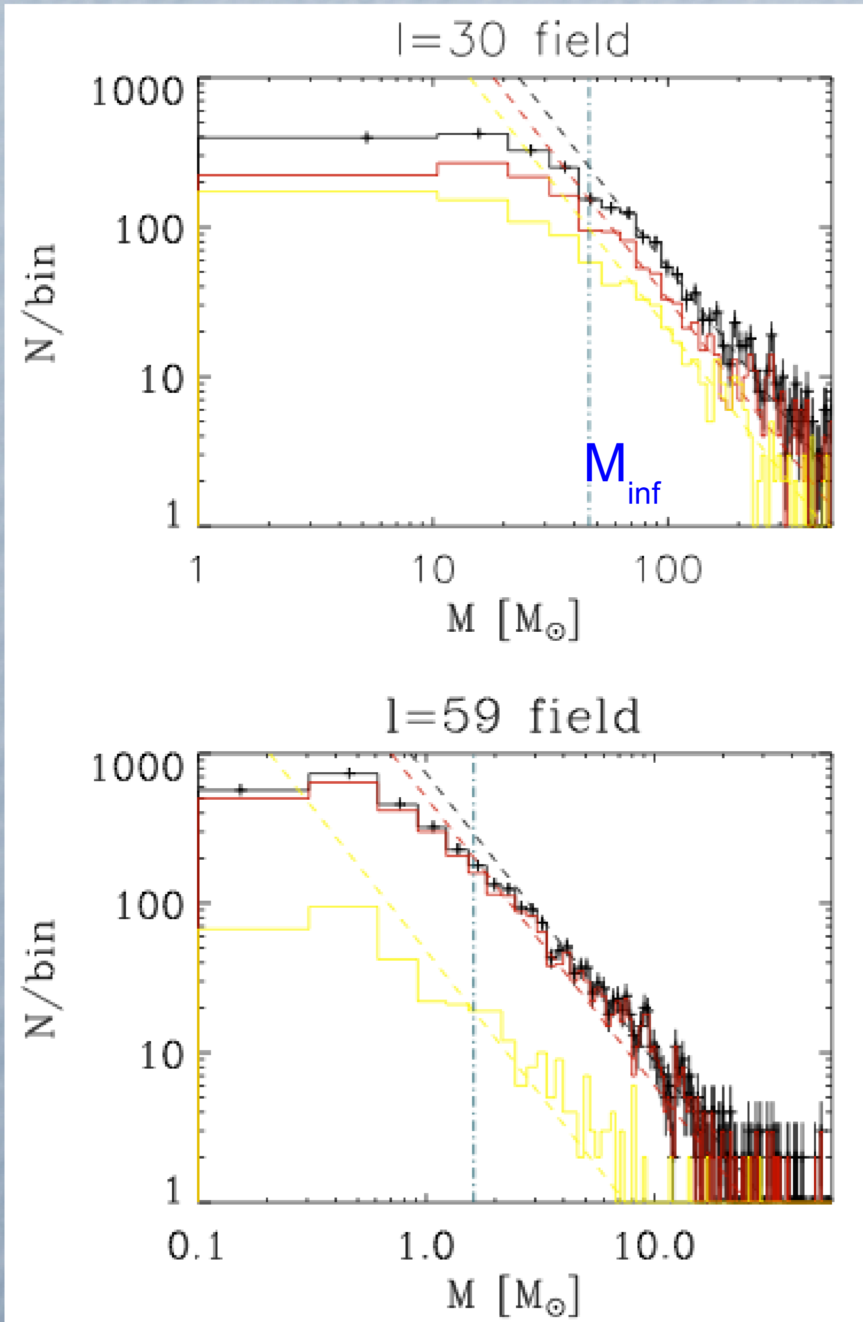
Hi-GAL data: l=30° and l=59° fields







Frequentist approach: powerlaw distribution



$$\xi(M) = \frac{dN}{dM} = \frac{\xi(\log M)}{M \ln 10} = \left(\frac{1}{M \ln 10}\right) \frac{dN}{d\log M}$$

$$\xi_{pw}(\log M) = A_{pw} M^{-\alpha}, \quad \text{or}$$

$$\xi_{pw}(M) = \frac{A_{pw}}{\ln 10} M^{-\alpha-1}.$$

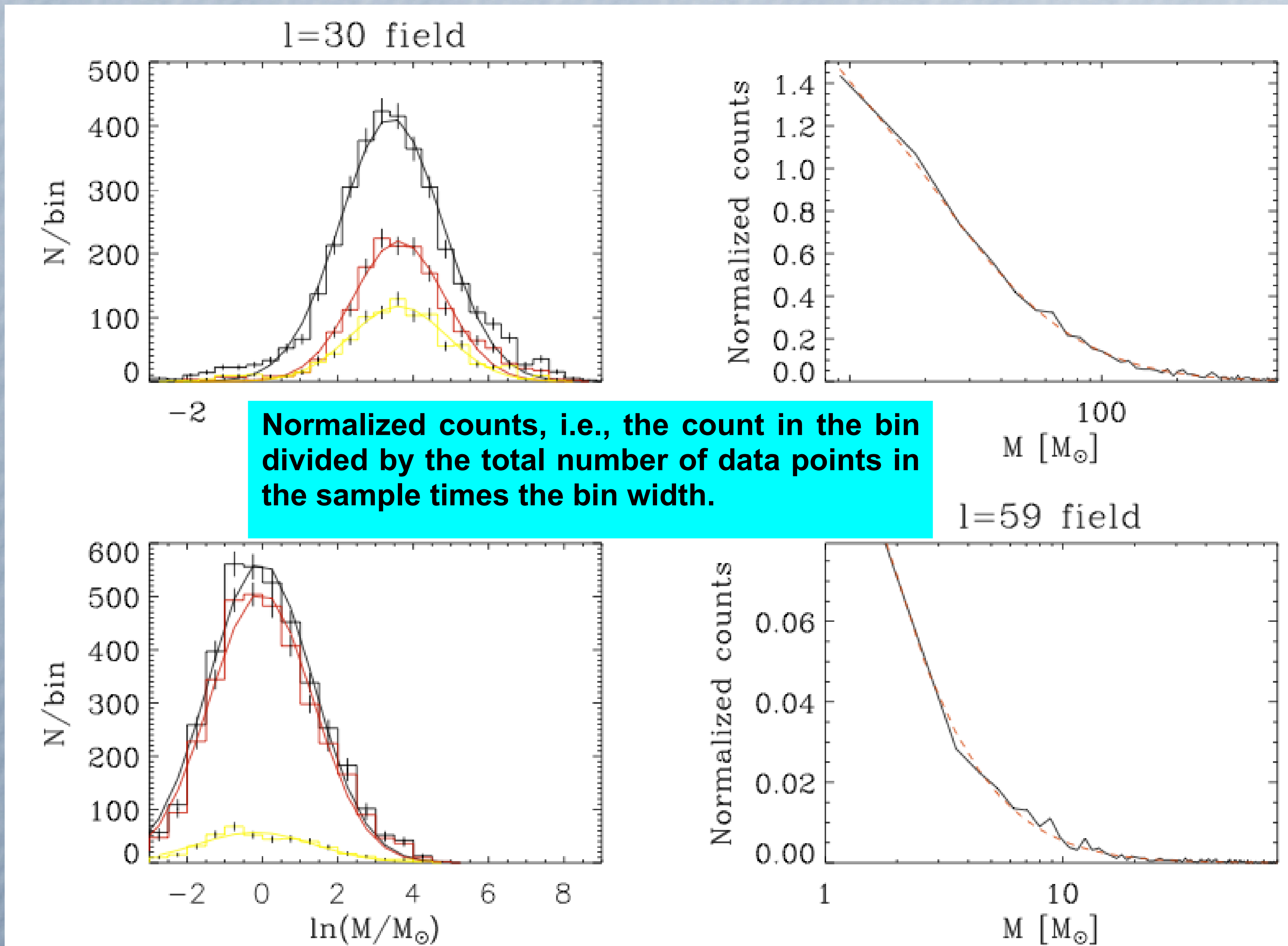
Generally valid if $M > M_{inf}$

We used a **MLE procedure** for fitting the powerlaw distribution to data, with a goodness-of-fit based approach to estimating the **lower cutoff, M_{inf}** .

(Clauset et al., 2009).

Population	$\ell = 30°$ **field**		$\ell = 59°$ **field**	
	α	M_{inf} $[M_\odot]$	α	M_{inf} $[M_\odot]$
All	1	46	0.9	1.6
Starless	1	46	0.9	1.6
Proto-stellar	1	46	1.0	2.8

Frequentist approach: log-normal distribution



Normalized counts, i.e., the count in the bin divided by the total number of data points in the sample times the bin width.

Frequentist approach:

$$\xi_{\ln}(\ln M) = \frac{A_{\ln}}{\sqrt{2\pi}\,\sigma}$$

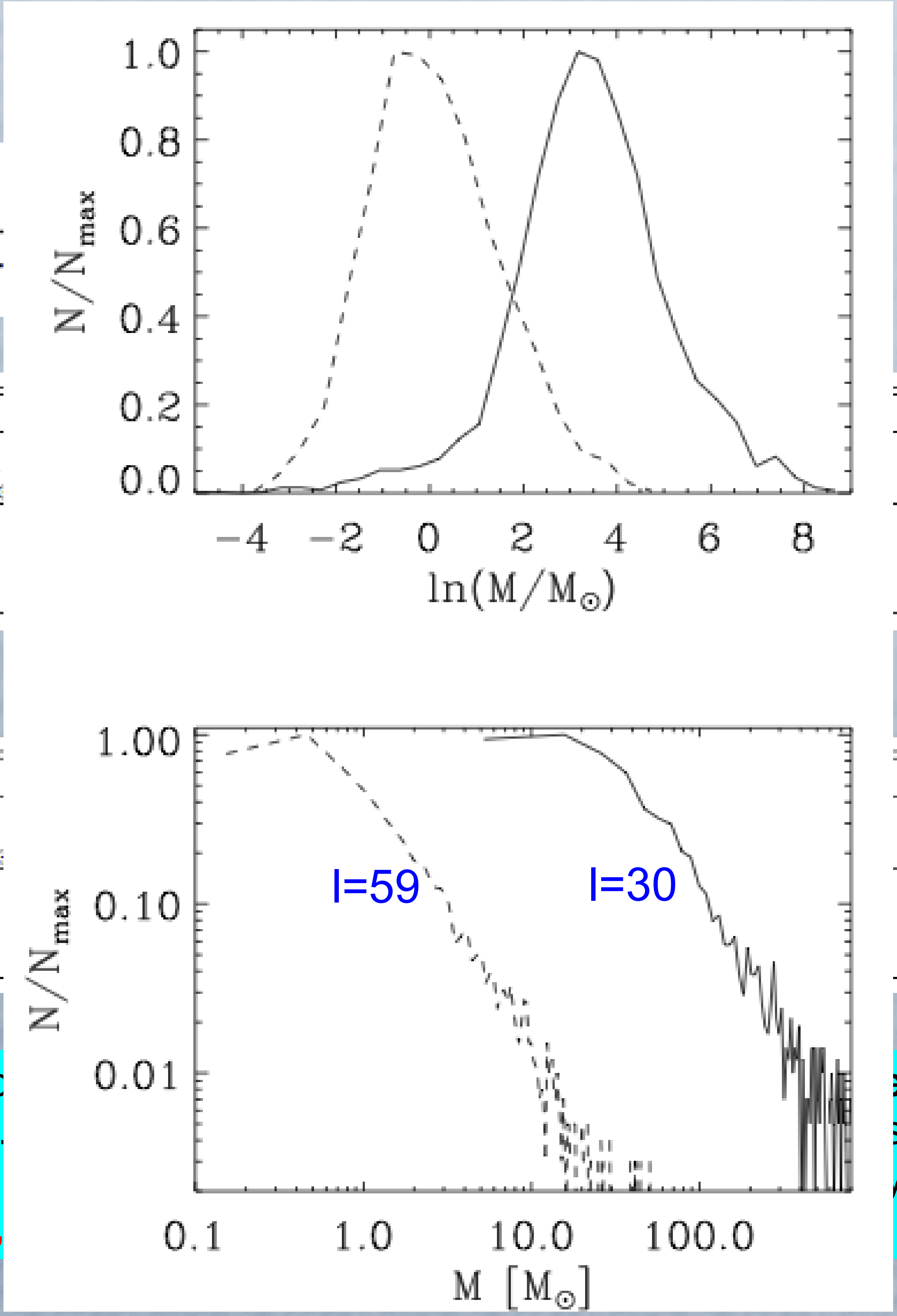
l=30

Population	Histogram best-fit		PDF best-fit	
	μ [ln $M_\odot$]	σ [ln $M_\odot$]	μ [ln $M_\odot$]	σ [ln $M_\odot$
All	3.4	1.4	3.4	1.3
Starless	3.6	1.2	–	–
Proto-stellar	3.6	1.3	–	–

l=59

Population	Histogram best-fit		PDF best-fit	
	μ [ln $M_\odot$]	σ [ln $M_\odot$]	μ [ln $M_\odot$]	σ [ln $M_\odot$
All	-0.04	1.4	0.19	1.5
Starless	-0.03	1.4	–	–
Proto-stellar	-0.18	1.5	–	–

The values of **μ are clearly different** fo… **remarkably similar**, i.e., [σ/ ln(*M*⊙)] ~ 1.… representing the distribution of the ln(*M*) … decidedly different mass scales (~30 factor),…



Bayes' Theorem

$$p(H|D,I) \propto p(H|I) \times p(D|H,I)$$

posterior ∝ prior × likelihood

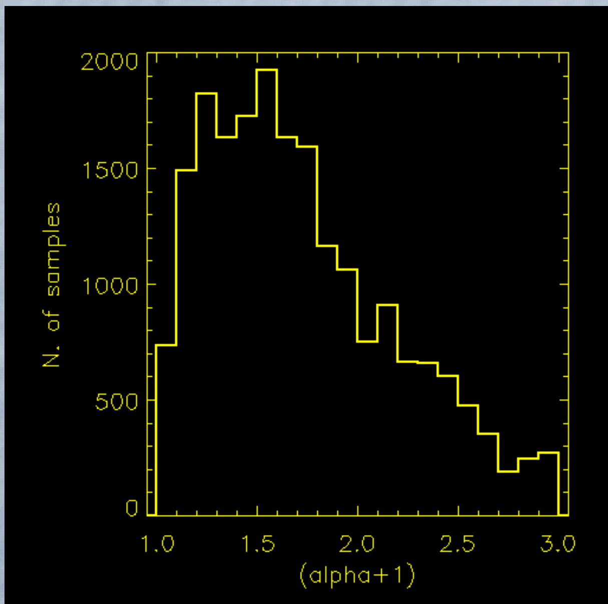
H = proposition asserting the truth of a **hypothesis** (could be a parameter or a model) of interest
I = proposition representing our **prior** information
D = proposition representing **data**

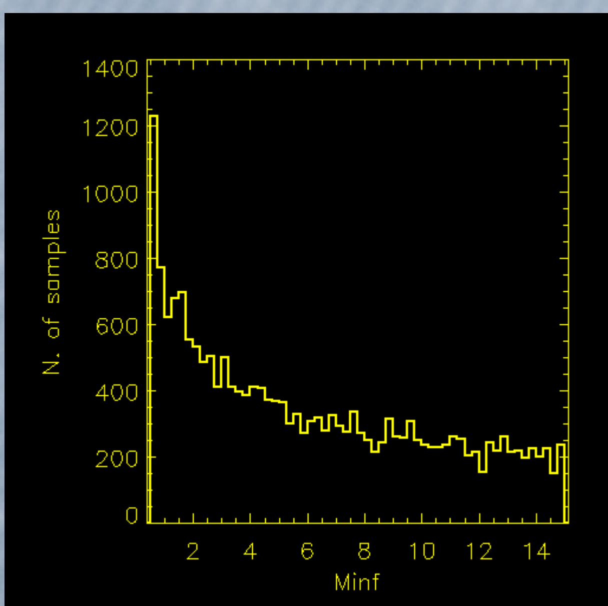
p(D|H,I) = probability of obtaining data D if H and I are true (also called the **likelihood** function *L(H)*)
p(H| I) **prior** probability of hypothesis
p(H|D,I) **posterior** probability of H

The Bayesian solution to the parameter estimation problem is the full posterior PDF, and not just a single point in parameter space. It is useful to summarize this distribution in terms of a "best-fit" value and "error bars."

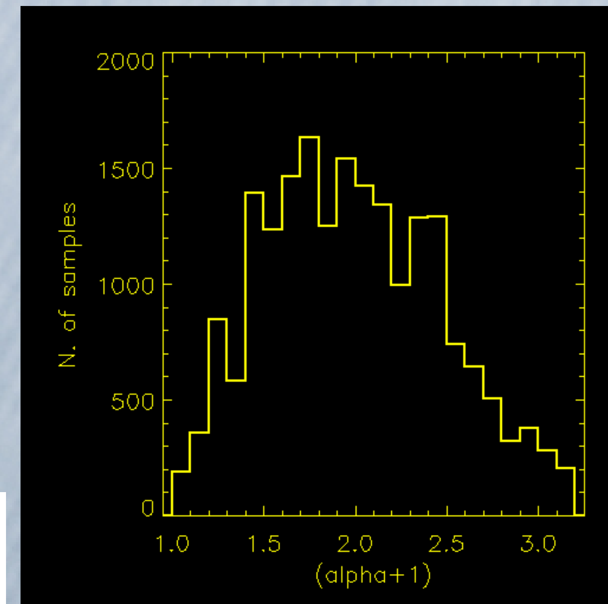
Bayesian approach: powerlaw distribution ($l=30^{\circ}$)

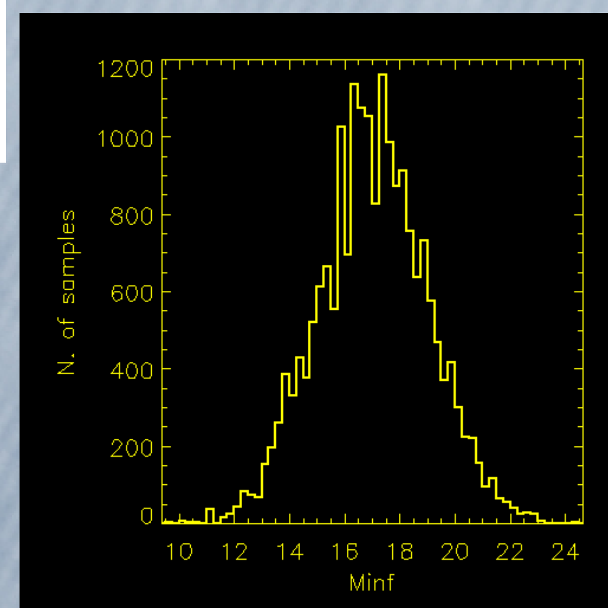
Jeffrey's Priors
α=0.6+/-0.3, Minf=5.1+/-(3.0/2.2) $M_{\odot}$





Gaussian Priors
α=0.9+/-0.3, Minf= 17.0+/-0.9 $M_{\odot}$



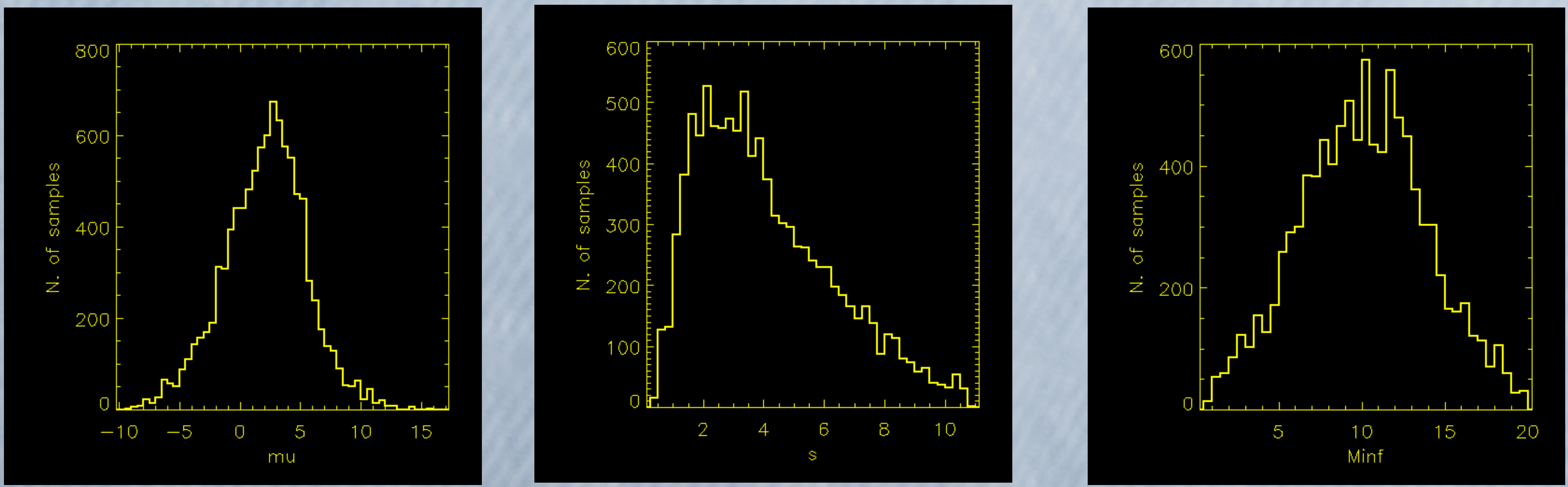


"Frequentist" Estimates

Population	$\ell = 30^{\circ}$ field		$\ell = 59^{\circ}$ field	
	α	$M_{\rm inf}$ $[M_{\odot}]$	α	$M_{\rm inf}$ $[M_{\odot}]$
All	1	46	0.9	1.6
Starless	1	46	0.9	1.6
Proto-stellar	1	46	1.0	2.8

Bayesian approach: <u>log-normal</u> distribution (l=30°)

Gaussian Priors



$\mu/\ln(M_\odot)$ = 2.3+/-(1.4/1.6) $\sigma/\ln(M_\odot)$ = 3.6+/-(1.3/1.0) $M_{inf}(M_\odot)$ = 10.0+/-1.9

"Frequentist" Estimates

Population	Histogram best-fit		PDF best-fit		MLE[a]		MLE with KS			
	μ [ln $M_\odot$]	σ [ln $M_\odot$]	μ [ln $M_\odot$]	σ [ln $M_\odot$]	μ [ln $M_\odot$]	σ [ln $M_\odot$]	μ [ln $M_\odot$]	σ [ln $M_\odot$]	M_{inf} [$M_\odot$]	M_{sup} [$M_\odot$]
All	3.4	1.4	3.4	1.3	4.0	1.9	3.7	1.9	12	103
Starless	3.6	1.2	–	–	–	–	–	–	–	–
Proto-stellar	3.6	1.3	–	–	–	–	–	–	–	–

CONCLUSIONS

- CMFs of the two Hi-GAL fields are quite similar in shape but with different mass scales: distance effect?

- Both CMFs show turn-over at lower-mass end, with different scales. Is M_{inf} region-dependent?

- A log-normal CMF can better fit the mass range $M<M_{inf}$

- No significant deviation from a powerlaw is observed at the higher-mass end

- Both frequentist and Bayesian techniques result in somewhat different parameters

- Bayesian approach to model selection is being analyzed