CASA Polarization Capabilities & Plans

George Moellenbrock (NRAO)

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Status

- Essential linear feed basis on-axis instrumental polarization calibration treatment supported and working
  - Calibrator $\mathcal{Q},\mathcal{U}$ estimation (from gains and from cross-hands)
  - Cross-hand phase *spectrum* estimation
  - Frequency-dependent Instrumental polarization solution (linearized, but including source polarization terms)
  - General matrix instrumental polarization correction
- Observational requirements (expectations):
  - Strongly (> few %) linearly polarized calibrator
  - Parallactic angle coverage sufficient for $\mathcal{Q},\mathcal{U}$ estimation (and therefore also for other polarization-specific steps)
- Simplified and degenerate approaches (e.g., unpolarized D calibrator, if available) also supported (position angle calibration for linears TBD)
- Pipeline support TBD

Status (cont)

- EVLA support
  - <1 GHz (linear feed basis): ALMA treatment applicable (polarized calibrators: pulsars)
  - >1 GHz (circular feed basis): traditional instrumental polarization treatment, with options for generalization (e.g., general D matrix correction)
    - Wider bandwidth==worse instr. pol.; dynamic range consequences
  - Ionospheric model corrections added in v4.3 (J. Kooi, UIowa)
    - A non-trivial example of 'forward' gain-like calibration

In the beginning (~2011)…

- Conditions:
  - Only polarized calibrators
  - No online XY-phase instrumentation
- ➢ Use strong and strongly polarized calibrators & sufficient parallactic angle coverage to solve for everything
  - Also, a polarized calibrator provides better constraints on an 'absolute' D solution for linear feeds
    - Full matrix correction, including parallel-hands (Stokes $\mathcal{I}$ dynamic range?)
  - How do the constraints actually work?
- ➢ For better or worse, this became the initial 'standard' observing mode (awkward in ALMA ops model which favors shorter observations)

CASA/ALMA Polarization Calibration Model

- Calibration Model:

$$V^{obs} = K^{crs}\, B^{r}\, G^{r}\, D^{r}\, X^{r}\, P\, V^{mod}$$

- Basic Solve sequence:
  - Normal bandpass (B^r) and gain (G^r) (parallel-hands):

$$V^{obs} = \underline{B^{r}}\, V^{\mathcal{I}}$$

$$(B^{r\,-1}\, V^{obs}) = \underline{G^{r}}\, V^{\mathcal{I}} \quad \text{(+estimate } \mathcal{QU}\text{)}$$

  - Cross-hand delay (K^{crs}), phase (X^r), and $\mathcal{Q},\mathcal{U}$ (cross-hands):

$$V^{obs} = \underline{K^{crs}}\, (B\, G\, P\, V^{\mathcal{IU}'})$$

$$(G^{r\,-1}\, B^{r\,-1}\, K^{crs\,-1}\, V^{obs}) = \underline{X^{r}}\, P\, V^{\underline{\mathcal{QUV}}} \quad \text{(+resolve amb)}$$

  - Revise G^r, using $\mathcal{IQU}$ (parallel-hands):

$$(B^{r\,-1}\, K^{crs\,-1}\, V^{obs}) = \underline{G^{r}}\, (P\, V^{\mathcal{IQU}})$$

  - Instrumental poln (D^r), using all of the above (cross-hands):

$$(G^{r\,-1}\, B^{r\,-1}\, K^{crs\,-1}\, V^{obs}) = \underline{D^{r}}\, (X^{r}\, P\, V^{\mathcal{IQU}})$$

  - (Iteration?)
- Correction:

$$V^{corr} = (P^{-1}\, X^{r\,-1}\, D^{r\,-1}\, G^{r\,-1}\, B^{r\,-1}\, K^{crs\,-1}\, V^{obs})$$

Ideal Visibilities: $\boldsymbol{V}^{\boldsymbol{true}}$

$$V_{XX} = \mathcal{I} + \mathcal{Q}$$

$$V_{XY} = \mathcal{U} + i\mathcal{V}$$

$$V_{YX} = \mathcal{U} - i\mathcal{V}$$

$$V_{YY} = \mathcal{I} - \mathcal{Q}$$

Parallactic Angle: $\boldsymbol{P\,V^{true}}$

$$V_{XX} = \mathcal{I} + (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} + \mathcal{Q}_\psi$$

$$V_{XY} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) + i\mathcal{V} = \mathcal{U}_\psi + i\mathcal{V}$$

$$V_{YX} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) - i\mathcal{V} = \mathcal{U}_\psi - i\mathcal{V}$$

$$V_{YY} = \mathcal{I} - (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} - \mathcal{Q}_\psi$$

$\boldsymbol{D}$ in the Linear Basis - I

$\boldsymbol{V = D\ P\ V^{true}}$:

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{U}_\psi+i\mathcal{V})d_{Xj}^* + d_{Xi}(\mathcal{U}_\psi-i\mathcal{V}) + d_{Xi}(\mathcal{I}-\mathcal{Q}_\psi)d_{Xj}^*$$

$$V_{XY} = (\mathcal{I}+\mathcal{Q}_\psi)d_{Yj}^* + (\mathcal{U}_\psi+i\mathcal{V}) + d_{Xi}(\mathcal{U}_\psi-i\mathcal{V})d_{Yj}^* + d_{Xi}(\mathcal{I}-\mathcal{Q}_\psi)$$

$$V_{YX} = d_{Yi}(\mathcal{I}+\mathcal{Q}_\psi) + d_{Yi}(\mathcal{U}_\psi+i\mathcal{V})d_{Xj}^* + (\mathcal{U}_\psi-i\mathcal{V}) + (\mathcal{I}-\mathcal{Q}_\psi)d_{Xj}^*$$

$$V_{YY} = d_{Yi}(\mathcal{I}+\mathcal{Q}_\psi)d_{Yj}^* + d_{Yi}(\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{U}_\psi-i\mathcal{V})d_{Yj}^* + (\mathcal{I}-\mathcal{Q}_\psi)$$

- Nice symmetries:
  - *d* multiplies pure cross-/parallel-hands in the parallel-/cross-hands
  - d^2 multiplies other pure parallel-/cross-hand (c.f. antenna-based description)

$\boldsymbol{D}$ in the Linear Basis - II

- Linearized, sorted:

$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{U}_\psi+i\mathcal{V})d_{Xj}^* + d_{Xi}(\mathcal{U}_\psi-i\mathcal{V})$

$V_{XY} = (\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{I}+\mathcal{Q}_\psi)d_{Yj}^* + d_{Xi}(\mathcal{I}-\mathcal{Q}_\psi)$

$V_{YX} = (\mathcal{U}_\psi-i\mathcal{V}) + d_{Yi}(\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{I}-\mathcal{Q}_\psi)d_{Xj}^*$

$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + d_{Yi}(\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{U}_\psi-i\mathcal{V})d_{Yj}^*$

$\boldsymbol{D}$ in the Linear Basis -III

- Linearized, sorted, $d\mathcal{V} \sim 0$, regrouped Stokes

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Xj}{}^*+d_{Xi})$$

$$V_{XY} = (\mathcal{U}_\psi+i\mathcal{V}) + \mathcal{I}\,(d_{Yj}{}^*+d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}{}^*-d_{Xi})$$

$$V_{YX} = (\mathcal{U}_\psi-i\mathcal{V}) + \mathcal{I}\,(d_{Yi}+d_{Xj}{}^*) + \mathcal{Q}_\psi\,(d_{Yi}-d_{Xj}{}^*)$$

$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Yi}+d_{Yj}{}^*)$$

  - Properties:
    - Constant (per baseline) complex offset proportional to $\mathcal{I}$ in cross-hands
    - *d*-scaled time-dependent source linear polarization in all correlations

Specific methods, modes

- `(almapolhelpers.py)`
- $\mathcal{Q},\mathcal{U}$ from naïve gain(t) ratio:
  - `qufromgain`
- XY-phase and $\mathcal{Q},\mathcal{U}$:
  - `gaincal(gaintype='XYf+QU')`
- XY-phase ambiguity resolution:
  - `xyamb`
- Frequency-dependent D solution ($\mathcal{Q}_\psi d$-sensitive):
  - `polcal(poltype='Dflls')`
- Sky frame D:
  - `dxy`
- General matrix correction:
  - `Dgen`

Gain calibration absorbs $\mathcal{Q},\mathcal{U}$

- Linear basis: If calibrator has non-zero (and unknown) linear polarization, polarization-dependent gain-like solves will absorb it, e.g.:

$$g_X' = g_X(1 + \mathcal{Q}_\psi/\mathcal{I})^{0.5} \quad g_Y' = g_Y(1 - \mathcal{Q}_\psi/\mathcal{I})^{0.5}$$

  - Parallel hands will be 'corrected' for *calibrator* polarization!
  - Cross-hand correction error is 2nd order in $\mathcal{Q}_\psi$: $(1 - \mathcal{Q}_\psi^2/\mathcal{I}^2)^{0.5}$
    - Distortion is small (we will rely on this later)
  - Formally, desirable to measure g_X, g_Y on an unpolarized source, depend on $|g_X/g_Y|$ stability, and use (unpol) $T(t)$
  - $\mathcal{Q},\mathcal{U}$ may be estimated if sufficient ψ sampling available, and true gain ratio is stable (`qufromgain`):

$$g_X'/g_Y' = (g_X/g_Y)\,(1 + \mathcal{Q}_\psi/\mathcal{I})^{0.5}/(1 - \mathcal{Q}_\psi/\mathcal{I})^{0.5}$$
$$\approx (g_X/g_Y)\,(1 - 2\mathcal{Q}_\psi/\mathcal{I})^{0.5}$$

Cross-hand phase spectrum

- An artifact of gain calibration reference antenna (refant)
- We do not measure absolute ***G*** and ***B***
- Instead, we measure $\boldsymbol{G^r}$ and $\boldsymbol{B^r}$, wherein a reference antenna's phase is fixed to zero in both polarizations, yielding relative phases for all other antennas
  - Differences among antennas in each polarization (*separately*) are preserved: no effect on parallel-hand calibration
- The refant's cross-hand bandpass phase remains undetected:

$$\boldsymbol{B\,G = B^r\,G^r\,X^r}$$

- And uncorrected:

$$\boldsymbol{G^{r\,-1}\,{}^{r}B^{r\,-1}\,B\,G = X^r}$$

$$\boldsymbol{X^r} = \begin{pmatrix} e^{i\rho} & 0 \\ 0 & 1 \end{pmatrix}$$

- $\boldsymbol{X^r}$ is as interesting as any bandpass phase spectrum in the system
  - (We are subject to its stability)

Revised factorization

- We therefore rewrite the calibration operator equation:

$$\boldsymbol{V^{obs} = B\, G\, D\, P\, V^{mod}}$$

$$\boldsymbol{= B^r\, G^r\, X^r\, D\, P\, V^{mod}}$$

- It is convenient to move $\boldsymbol{X_r}$ upstream of $\boldsymbol{D}$:

$$\boldsymbol{= B^r\, G^r\, D^r\, X^r\, P\, V^{mod}}$$

$$\boldsymbol{(D^r = X^r\, D\, X^{r-1})}$$

  - $\boldsymbol{D^r}$ is the instrumental polarization measured in the cross-hand phase frame of the gain & bandpass calibration reference antenna
- In the linear basis, $\boldsymbol{X^r}$ must be determined so cross- and parallel-hands can be combined to extract correct Stokes parameters
- In the circular basis, $\boldsymbol{X^r}$ is just a polarization position angle offset, which can be deferred for later external calibration

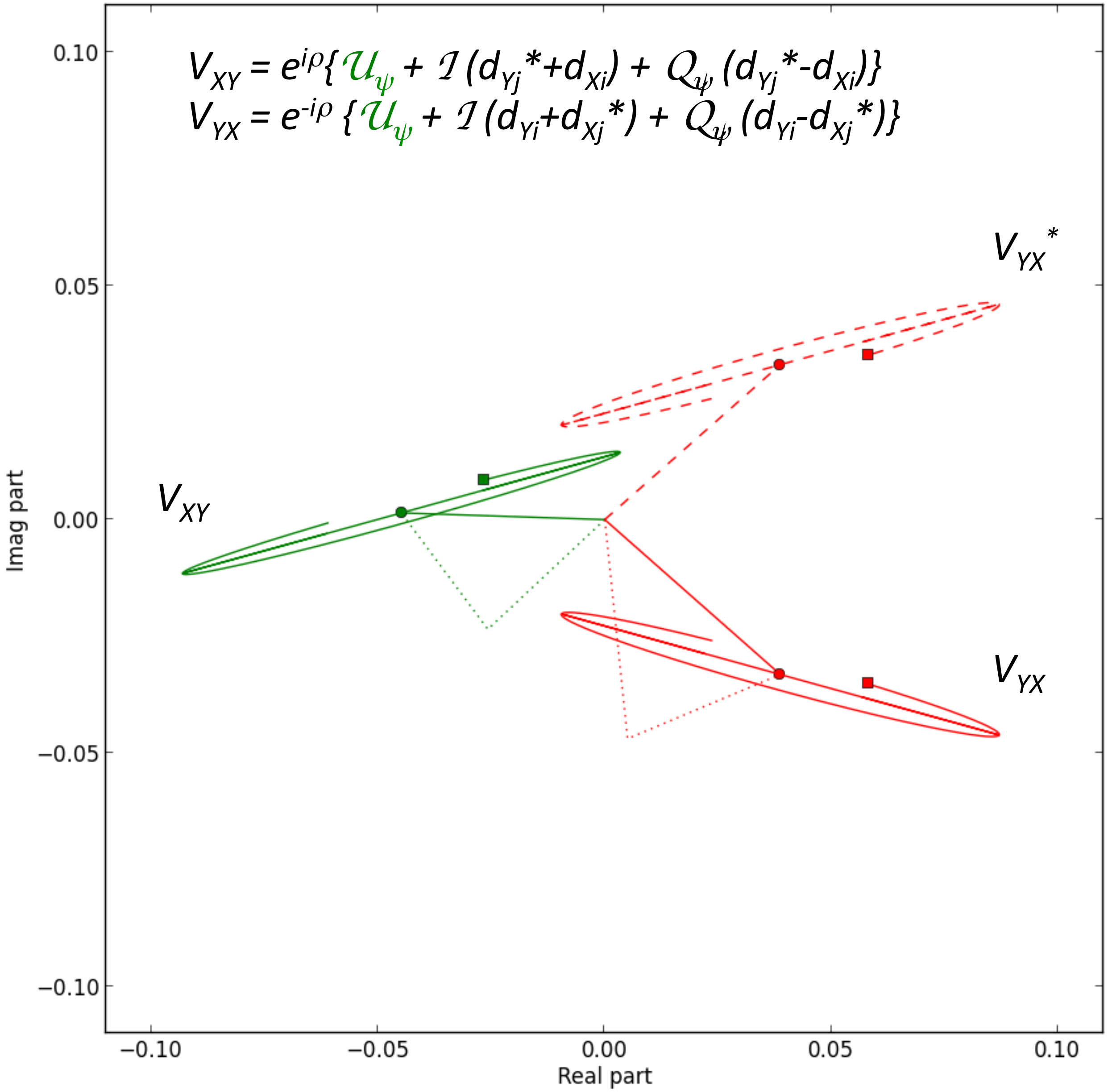
Solving for $\boldsymbol{X^r}$ and $\mathcal{Q},\mathcal{U}$

$$(G^{r\,-1}\, B^{r\,-1}\, V^{obs}) = X^r\, D\, P\, V^{mod}$$

- Consider just the gain- and bandpass-calibrated cross-hands:

$$V_{XY} = e^{i\rho}\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yj}{}^* + d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}{}^* - d_{Xi})\}$$

$$V_{YX} = e^{-i\rho}\,\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yi} + d_{Xj}{}^*) + \mathcal{Q}_\psi\,(d_{Yi} - d_{Xj}{}^*)\}$$



$V_{XY} = e^{i\rho}\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yj}{}^* + d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}{}^* - d_{Xi})\}$
$V_{YX} = e^{-i\rho}\,\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yi} + d_{Xj}{}^*) + \mathcal{Q}_\psi\,(d_{Yi} - d_{Xj}{}^*)\}$
$V_{YX}{}^*$
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Imag part
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Real part

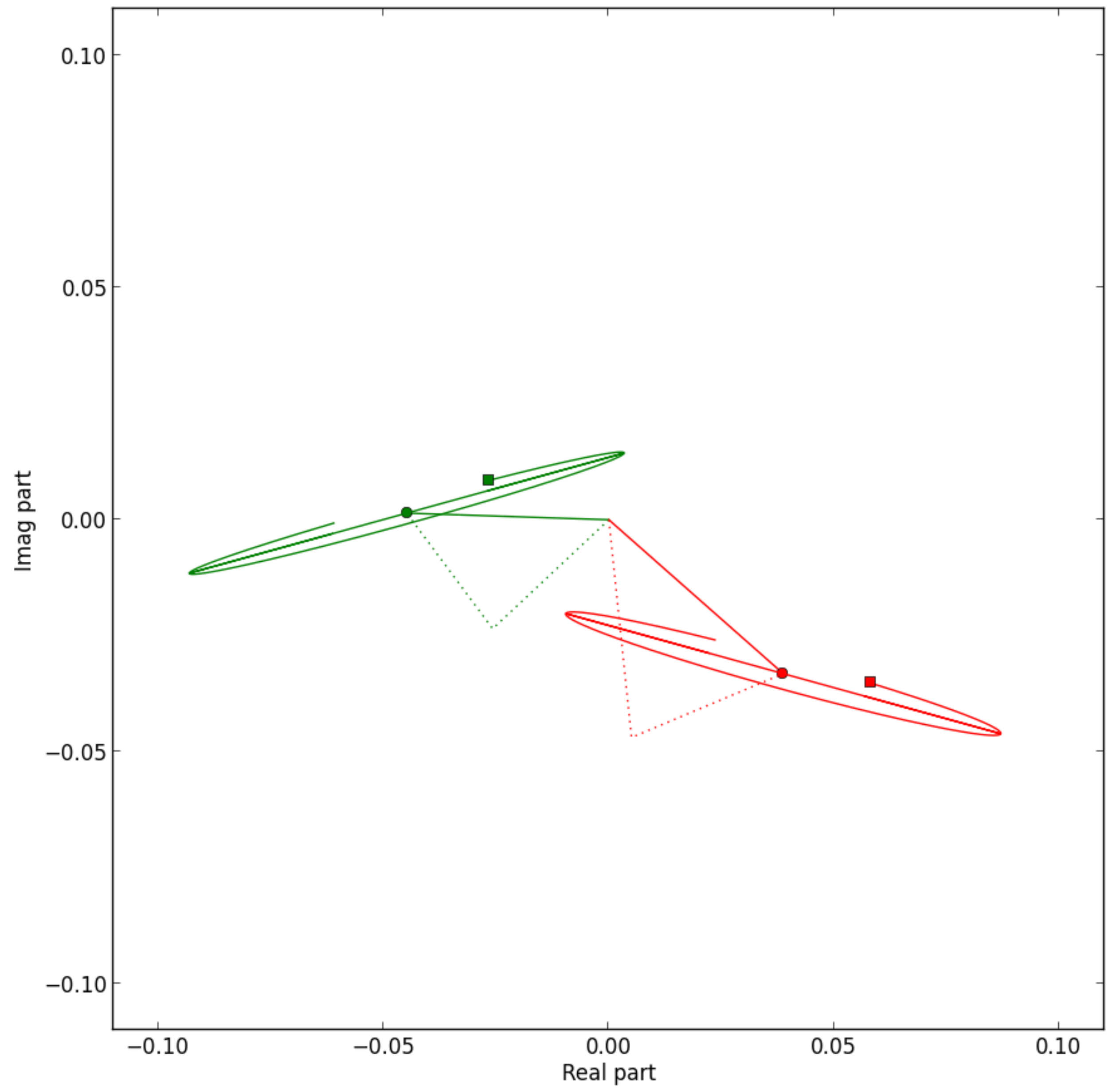
Solving for $\boldsymbol{X}^r$ and $\mathcal{Q},\mathcal{U}$ (cont)

- Average over baselines and correlations:

$$(\langle V_{XY}\rangle + \langle V_{YX}{}^*\rangle)/2 = \mathcal{U}_\psi e^{i\rho} + \langle \mathcal{I}(d_{Yj}{}^*+d_{Xi}) + \mathcal{Q}_\psi (d_{Yj}{}^*-d_{Xi})\rangle e^{i\rho}$$

$$= (-\mathcal{Q}\,sin2\psi + \mathcal{U}\,cos2\psi)e^{i\rho} + \varepsilon(t)$$

  - $\varepsilon(t)$ is small if *d'*s are small and ~random
  - Requires non-zero $\mathcal{Q}, \mathcal{U}$
  - Measurements at (at least) 3 distinct ψ sufficient to determine $\rho, \mathcal{Q}, \mathcal{U}, \varepsilon$
    - gaincal(gaintype='XYf+QU')
  - Ambiguity: $(\rho, \mathcal{Q}, \mathcal{U}) \rightarrow (\rho+\pi, -\mathcal{Q}, -\mathcal{U})$
    - Resolvable using $\mathcal{Q}, \mathcal{U}$ estimate from gain ratio: `xyamb`
  - Requires $\boldsymbol{X}^r$ stability: a "good" refant for gain and bandpass (c.f. $|g_X/g_Y|$ stability expectations)
  - ATCA (linears) use a calibration signal to monitor $\boldsymbol{X}^r$
    - refant operation in $\boldsymbol{G}^r$ and $\boldsymbol{B}^r$ solves will then merely enforce the truth)



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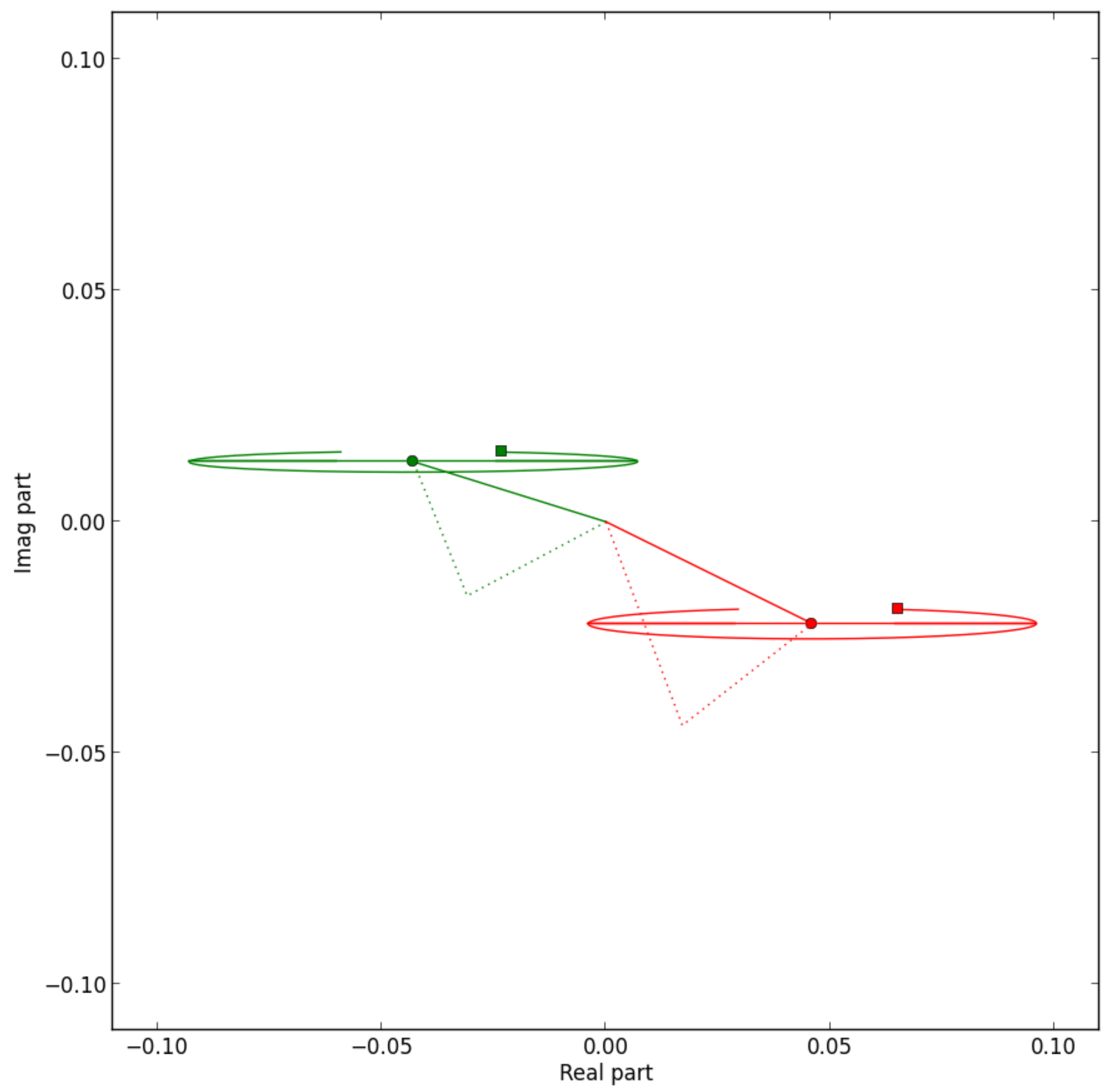

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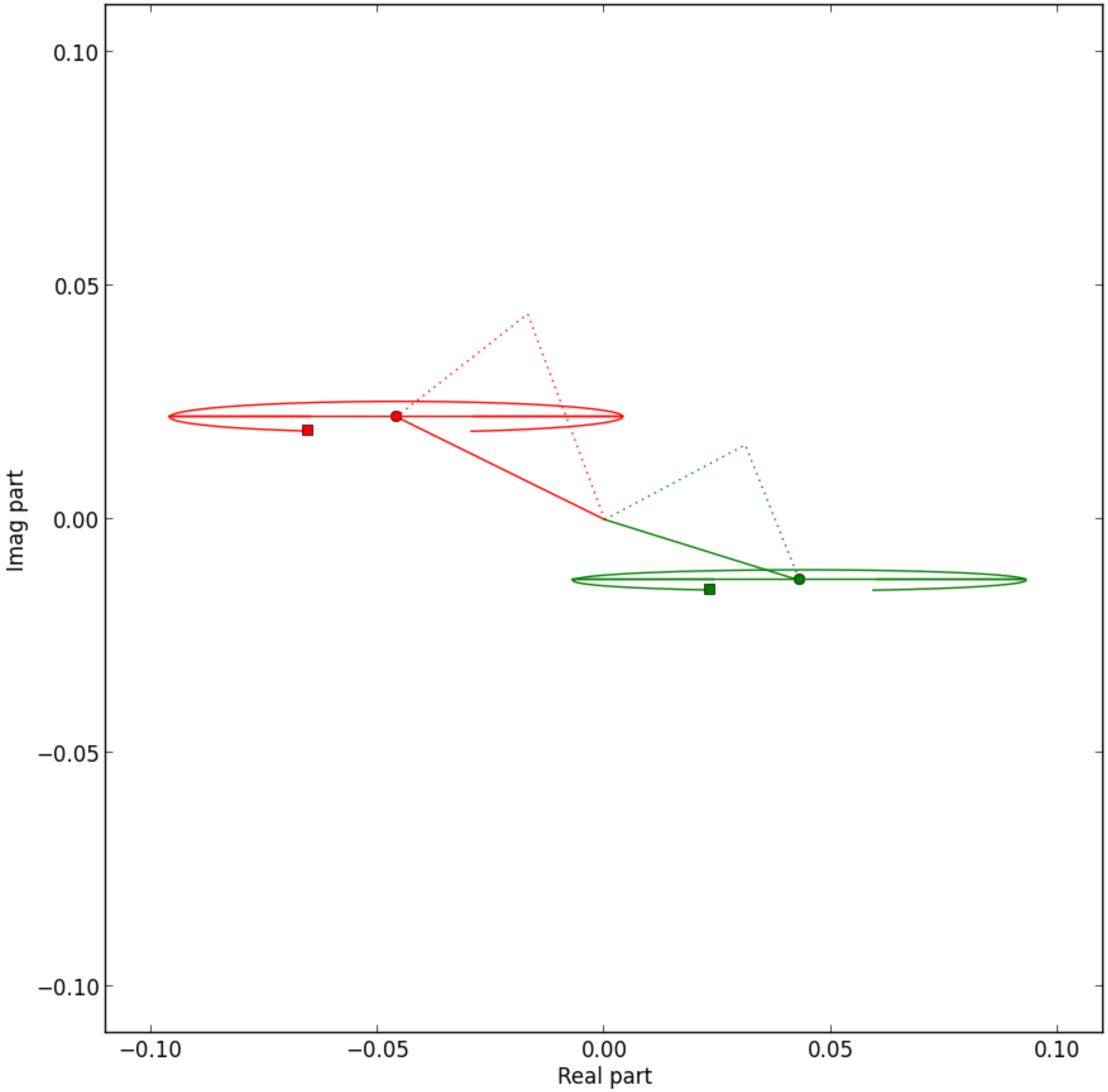

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Real part

D

- Frequency-dependent D solution ($\mathcal{Q}_\psi$*d*-sensitive): `polcal(poltype='Dflls')`
- Sky frame ***D***: `dxy`
  - Rotate out of gain refant cross-hand phase frame:
    - $\boldsymbol{D = (X^{r-1}\, D^r\, X^r)}$
  - Continuity over spectral windows
  - Appropriate frame for 'canned' ***D*** (not tied to a specific *gain* refant)
- General matrix correction: `Dgen`
  - Relabeling to enforce full matrix correction

Additional Topics

- Solving for ***D*** (linear basis): Unpolarized vs. Polarized Calibrator?
- Multi-field solution?
- Stokes $\mathcal{V}$?
- 'Canned' ***D***?
- Poorly-optimized optics?

Solving for ***D*** (linear basis)

- Unpolarized source, single scan?
  - Simple, but *d's are* degenerate, to first order:

    $V_{XY} = \mathcal{I}(d_{Yj}^{*}+d_{Xi}) = \mathcal{I}[(d_{Yj}-a)^{*}+(d_{Xi}+a^{*})]$

    - for any complex number *a*
  - Therefore, one *d* remains effectively unconstrained
  - Standard convention is to apply a reference antenna that effectively enforces $a = -d_{Xref}^{*}$:

    $d_{xi} \rightarrow (d_{Xi}+a^{*}) \quad d_{Yi} \rightarrow (d_{Yi}-a)$ (for all *i*)
  - These referenced *d*'s correct the data to some *orthogonal* (to first order) basis that is defined by the refant's true d_X-- but not a *pure* one
  - Position angle calibration satisfies refant's real part
    - Imag part (ellipticity registration)?

Solving for ***D*** (linear basis)

- Polarized (known) source, single scan ($\psi = const$)?
  - Still degenerate in linear cross-hands (incl. $\mathcal{Q}_\psi$ terms):

$$V_{XY} = \mathcal{U}_\psi + (\mathcal{I} + \mathcal{Q}_\psi) d_{Yj}{}^* + (\mathcal{I} - \mathcal{Q}_\psi) d_{Xi}$$
$$= \mathcal{U}_\psi + (\mathcal{I} + \mathcal{Q}_\psi)(d_{Yj} - b)^* + (\mathcal{I} - \mathcal{Q}_\psi)(d_{Xi} + c^*)$$

    - for any complex numbers *b, c* satisfying

$$b = c(\mathcal{I} + \mathcal{Q}_\psi)/(\mathcal{I} - \mathcal{Q}_\psi)$$

  - Incorrectly calibrates data with different $\mathcal{Q}_\psi$ (other times, sources)
  - If $\mathcal{Q}_\psi d$ terms ignored, same as unpolarized source case

Solving for ***D*** (linear basis)

- Polarized source w/ parallactic angle coverage
  - Multiple, non-zero $\mathcal{Q}_{\psi}$ breaks degeneracies
  - Depends on time stability of ***D***
    - also required/assumed for accurate transfer to other sources
  - Resulting solution accuracy limited by:
    - accuracy of calibrator model's $\mathcal{Q},\mathcal{U},\mathcal{V}$
    - systematic bias (if any) in assumed feed position angle setting (if $\mathcal{Q},\mathcal{U}$ derived from data)
    - systematic biases (if any) originating in 2^{nd}-order terms

Multi-field *D* solution?

- Each new source adds 2 unknowns, with insufficient additional constraints to determine them
  - Still need at least one source with sufficient parallactic angle coverage
- VLBI (circular basis) leverages differential parallactic angle variation to constrain 'absolute' ***D***
  - ALMA is a 'small array' with practically uniform parallactic angle across the array

Stokes $\mathcal{V}$?

- Spectral line (e.g., Zeeman)
  - Enabled by detecting specified spectral signature relative to arbitrary background "continuum/ systematic" level, which is fit for but ignored
- Continuum
  - Effectively unconstrained without *absolute* $\mathcal{V}$ reference
  - Statistical constraint: universe has no preferential handedness (e.g., ATCA: Rayner *et al.*, 2000)

‘Canned’ ***D***?

- Yes, if reliably stable!
  - In CASA:
    - Requires formal recognition of ‘sky-frame’ ***D***’s in the internal calibration model (TBD)
    - Requires look-up mechanism (c.f. antenna-position corrections, etc.)
    - General matrix algebra?
  - At ALMA (and EVLA): Requires measuring it adequately (incl. frequency coverage at sufficient resolution)
    - Updates required at RX ‘events’ (c.f. antenna-position corrections when telescopes move)
  - Natural extension of beam polarization models?
    - Add, don’t multiply!
    - Not as simply modeled, probably?

Non-optimized optics?

- Effect of poor feed mount collimation?
- Vertex astigmatism?
- Effect of tilt-less sub-reflector?
  - *Primary* optics effectively asymmetric
  - (c.f. beam models?)
- Coupled effects, e.g. focus and pointing models are deliberate time-dependent perturbations on the optics
- 'Forward' polarization-dependent gain effects?

➢ Effects on ***D*** and its stability?

CASA Planning

- The following is George's personal view
- NB: All polarization development plans must be evaluated and prioritized alongside general CASA development cycle planning
  - V4.4 development underway; release: ~Apr 2015
  - V4.5 planning beginning now; release: ~Oct 2015
- Pipeline context…

CASA Todo List - I

- Algorithm/Robustness Improvements
  - Position angle calibration for linear basis (needed for unpolarized D calibrator case)
    - Ellipticity registration calibration?
  - Support "Sky-frame" ***D*** in general apply
    - Enables canned ***D***
  - $\mathcal{Q},\mathcal{U}$ model storage/usage (MS.SOURCE) to streamline processing
    - Eases migration of most `almapolhelpers.py` functionality into standard CASA tasks
  - qufromgain: option to remove source polarization signal from gains
    - Also detect/remove antennas with outlying gain ratios; antenna (de-)selection?
  - Iteration packaging (has been deliberately 'naked', for now, but only minimally explored so far)
- Modularization
  - Separate (optionally) XY-phase and QU estimation
    - Ad hoc XY-phase ambiguity switch (gencal)
  - Migrate 'Xyf+QU' functionality from gaincal to polcal

CASA Todo List - II

- Improve Documentation!
  - CASAguide by Rosita
  - Memos
- Visualization
  - Viewer enhancments (e.g., simplified vector plotting)
  - Stokes options in plotms (flagging?)
- Simulation
  - Corruption by full magnitude instrumental polarization
  - Corruption by plausible residual instrumental polarization (so as to yield realistically ‘poor’ result)