Short intro to Interferometry

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ALMA Regional Centre || Italian

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Ideas and slides borrowed from

IRAM interferometry school

http://www.iram-institute.org/EN/content-page-331-7-67-331-0-0.html

NRAO interferometry school

https://science.nrao.edu/science/meetings/2016/15th-synthesis-imaging-workshop

LOFAR school

http://www.astron.nl/lofarschool2016/

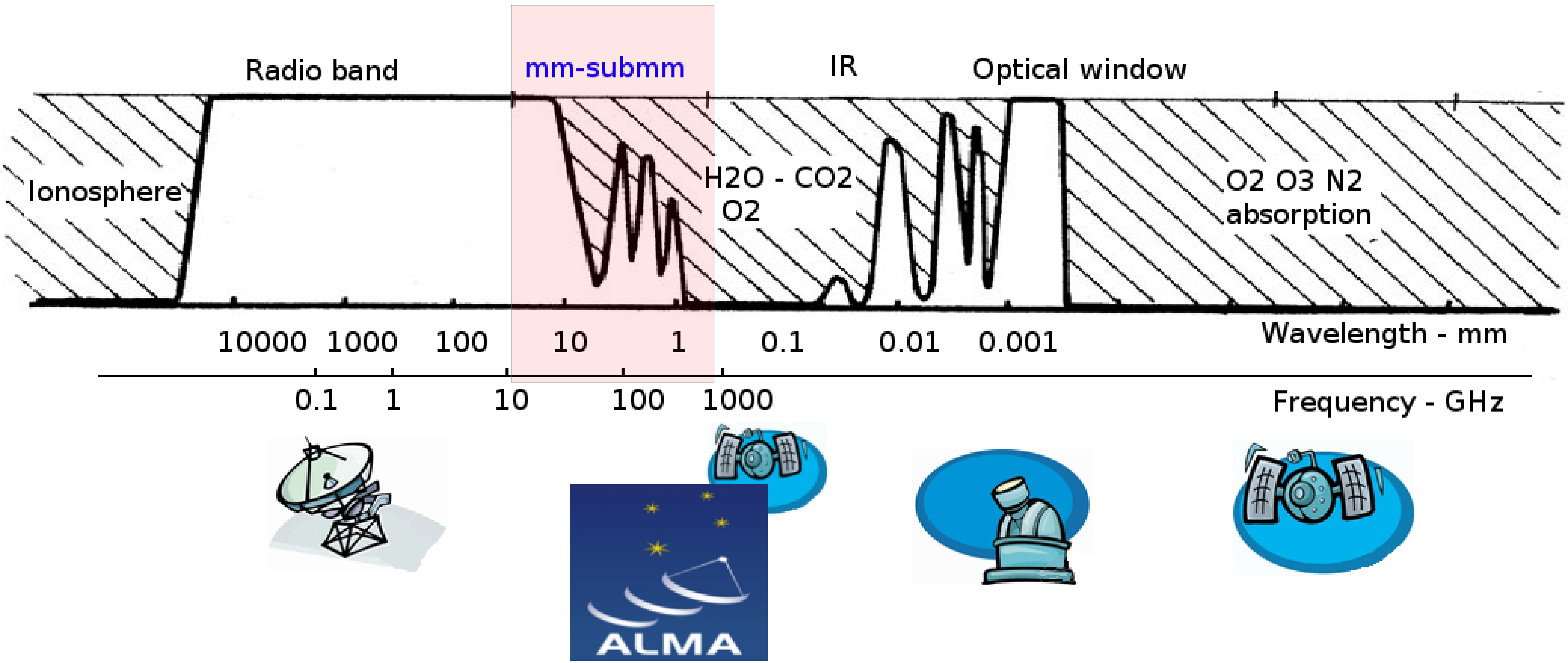
European Radio interferometry (ERIS) school

https://www.eso.org/sci/meetings/2015/eris2015.html

Synthesis Imaging in Radio Astronomy: II – The “White Book”

Virtual Radio Interferometer
http://www.narrabri.atnf.csiro.au/astronomy/vri.html

submm interferometer....



~ 80-900 GHz
~ 3-0.3 mm

Resolution of Observations

Angular resolution for most telescopes is ~ λ/D

- D is the diameter of the telescope
- λ is wavelength of observation

For example, Hubble Space Telescope:

- $\lambda \sim 1$um / D of 2.4m = resolution ~ 0.13"

To reach that resolution for a λ ~1mm observation, one would need a 2 km-diameter dish!

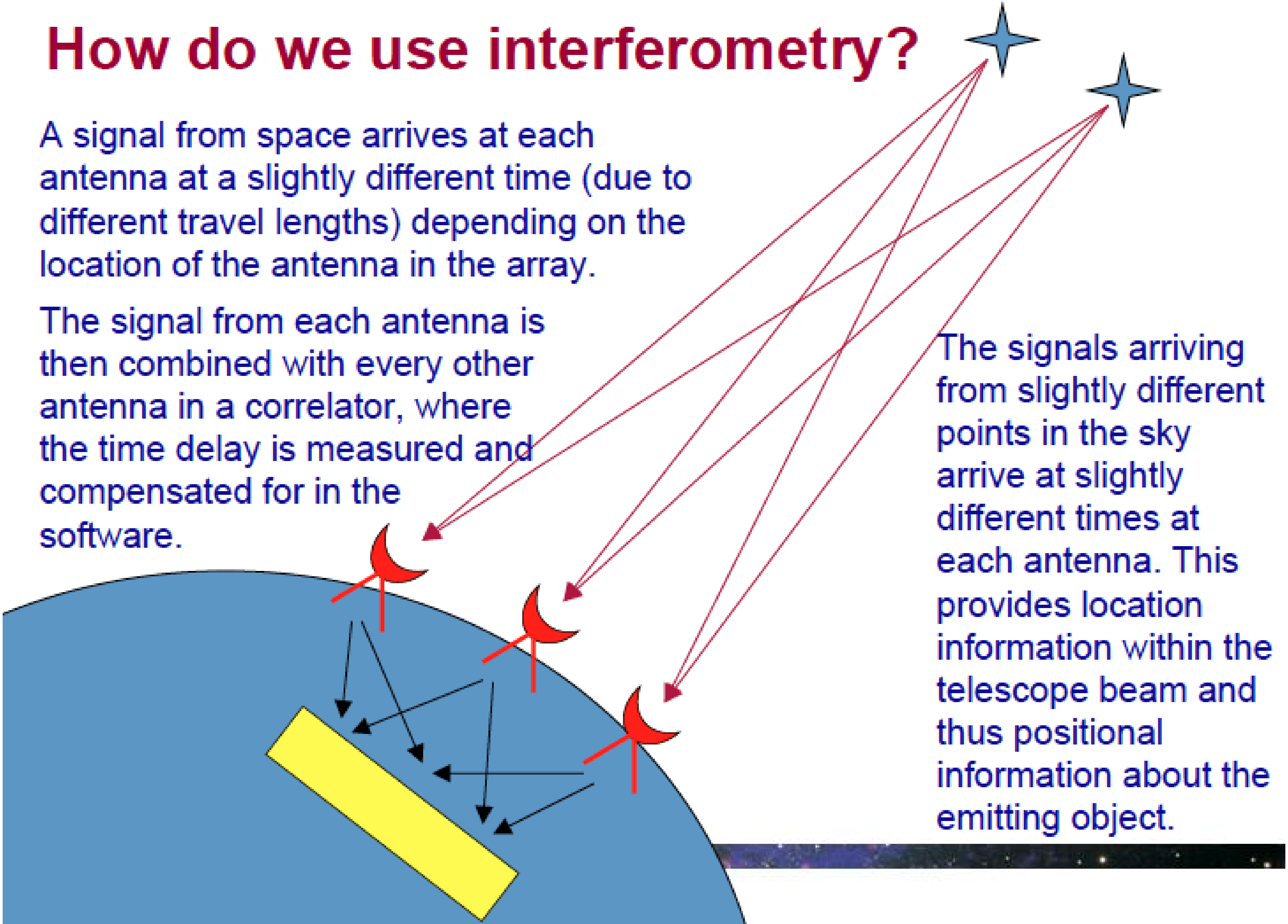
Instead, we use arrays of smaller dishes to achieve the same high angular resolution at radio frequencies

This is interferometry

How do we use interferometry?

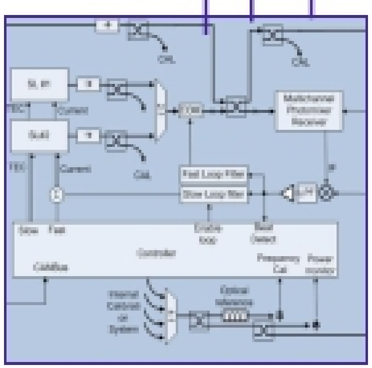
A signal from space arrives at each antenna at a slightly different time (due to different travel lengths) depending on the location of the antenna in the array.

The signal from each antenna is then combined with every other antenna in a correlator, where the time delay is measured and compensated for in the software.



The signals arriving from slightly different points in the sky arrive at slightly different times at each antenna. This provides location information within the telescope beam and thus positional information about the emitting object.

Some instrument details…

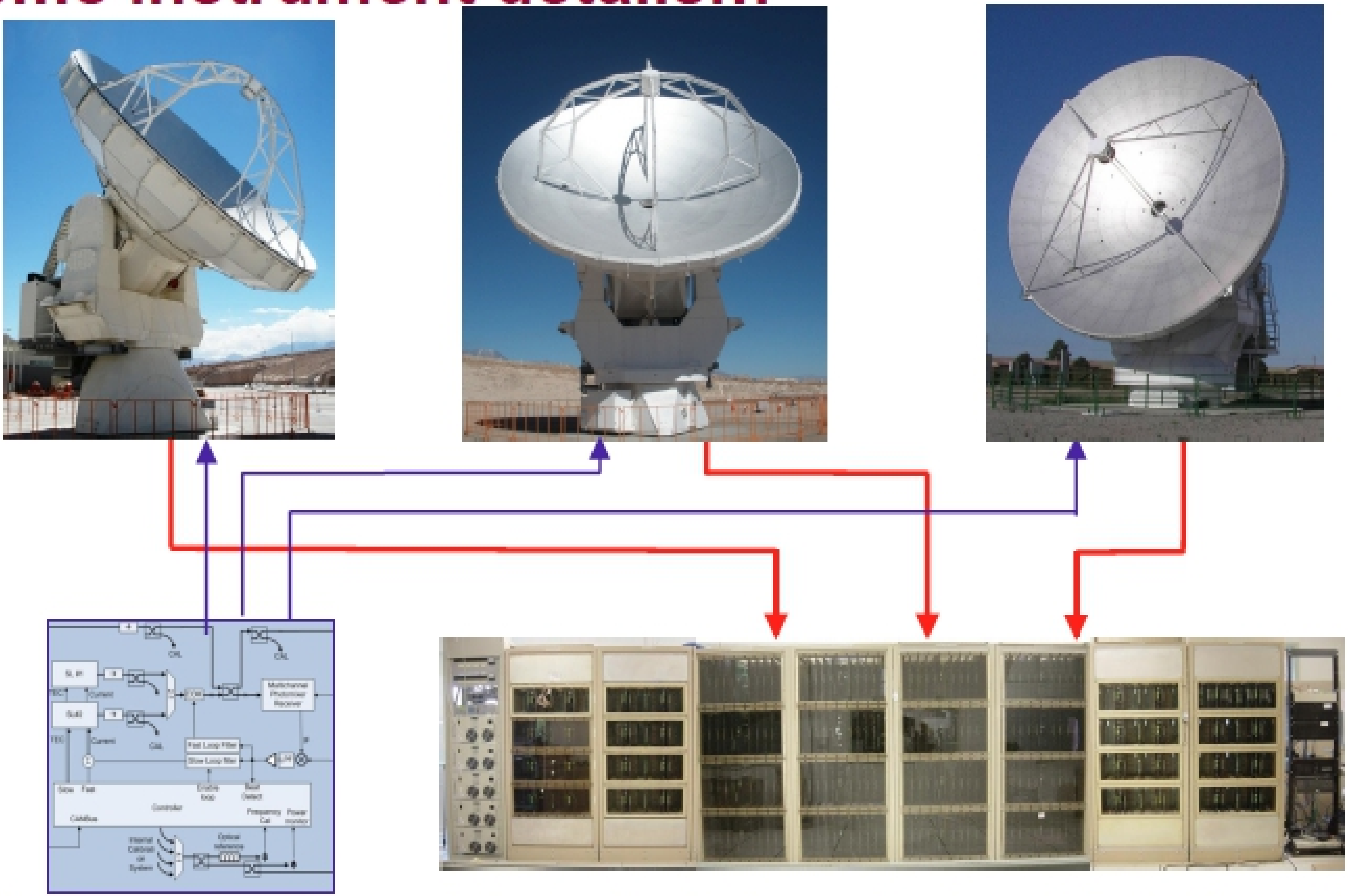


To precisely measure arrival times we need very accurate clocks

- At Band 10 one wavelength error = 1 picosecond (!!)
- We need << 1 wavelength timing precision so each antenna has an on-board clock with high sampling rates

Once determined, the reference time is distributed to all antennas

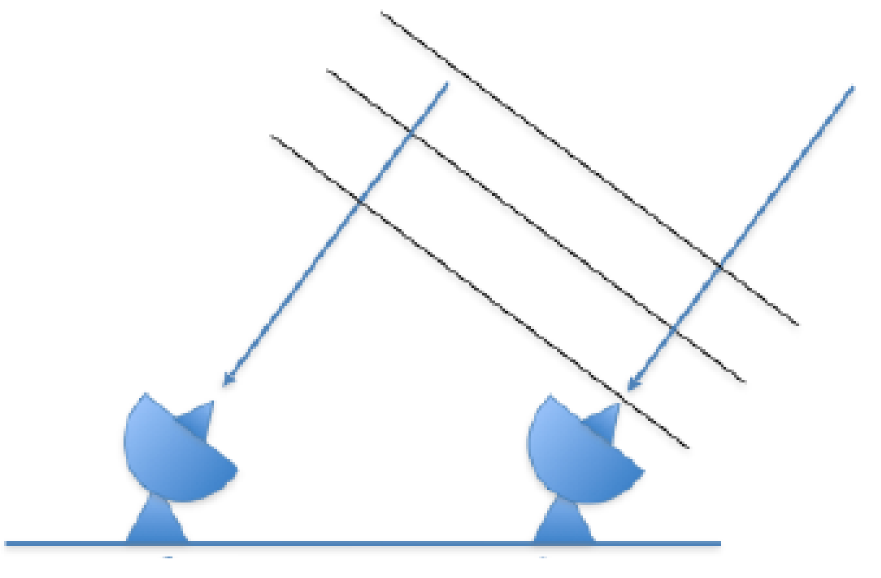
Some instrument details…



Signal from each antenna are digitized and sent to the correlator for multiplication & averaging.
For ~50 antennas the data rate is 600 GB/sec for the correlator to process

Interferometers: the basics

- Interferometry: a method to 'synthesize' a large aperture by combining signals collected by separated small apertures
- An Interferometer measures the interference pattern produced by two apertures, which is related to the source brightness.
- The signals from all antennas are correlated, taking into account the distance (baseline) and time delay between pairs of antennas

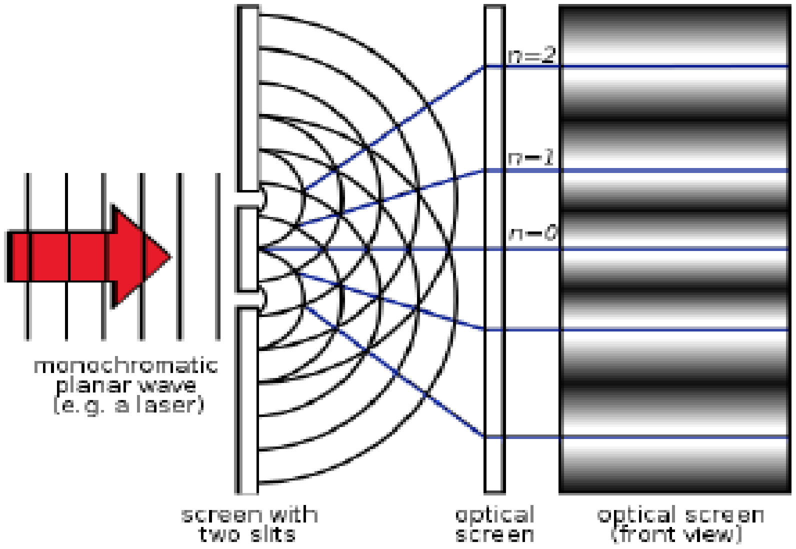


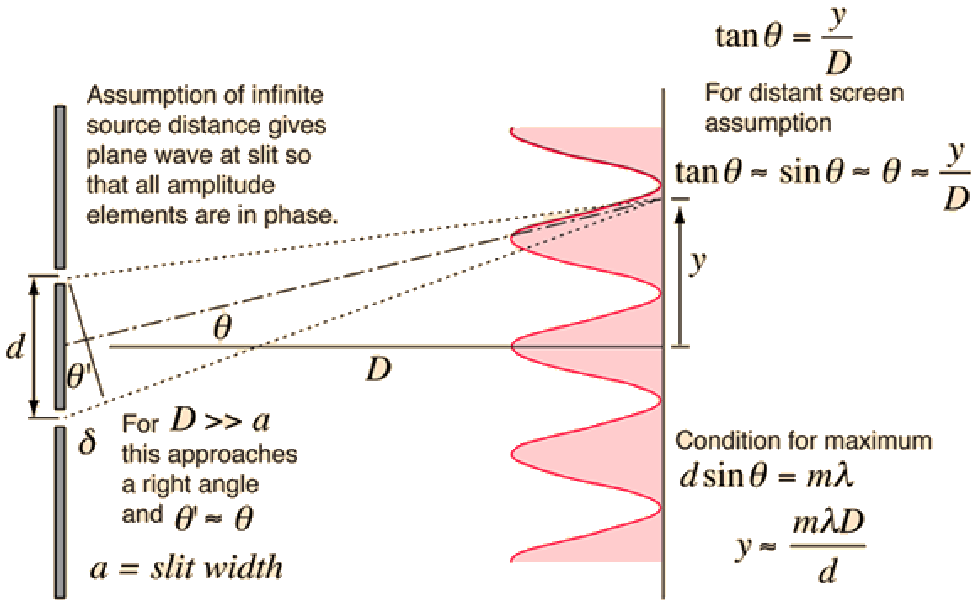
$\Theta \sim \lambda/D$

D

which is related to b - baseline

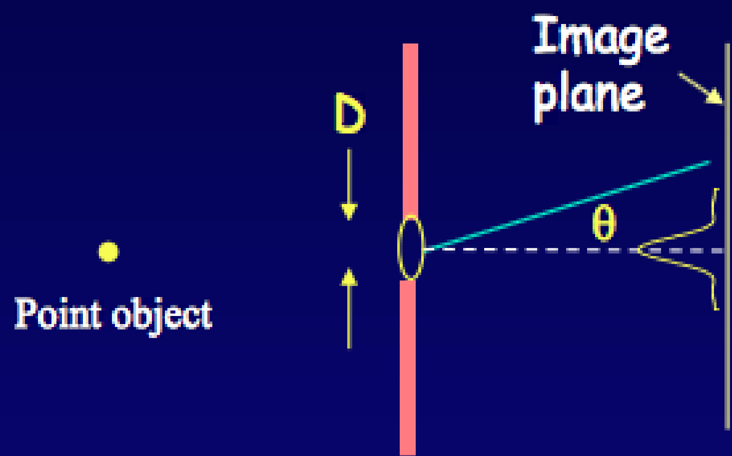
much like a 2-slit experiment

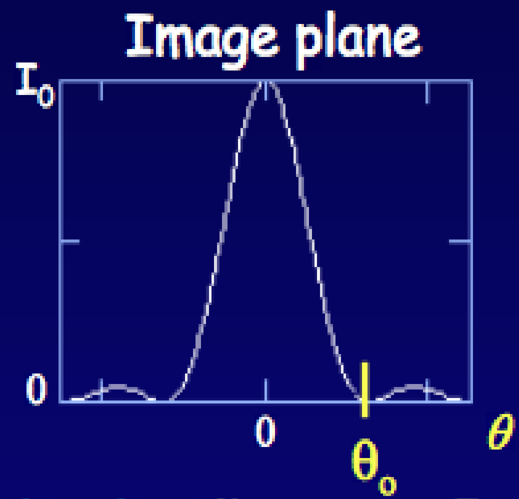


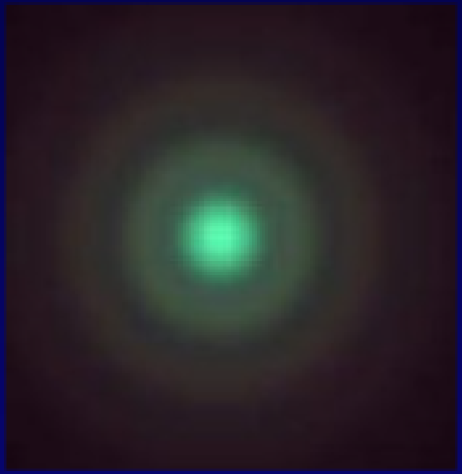


Diffraction has important implications for optical instruments

Even for perfectly designed optics the image of a point source will be a little blurry - the circular aperture produces diffraction.







The "Airy disk".
The central lobe contains 84% of power.

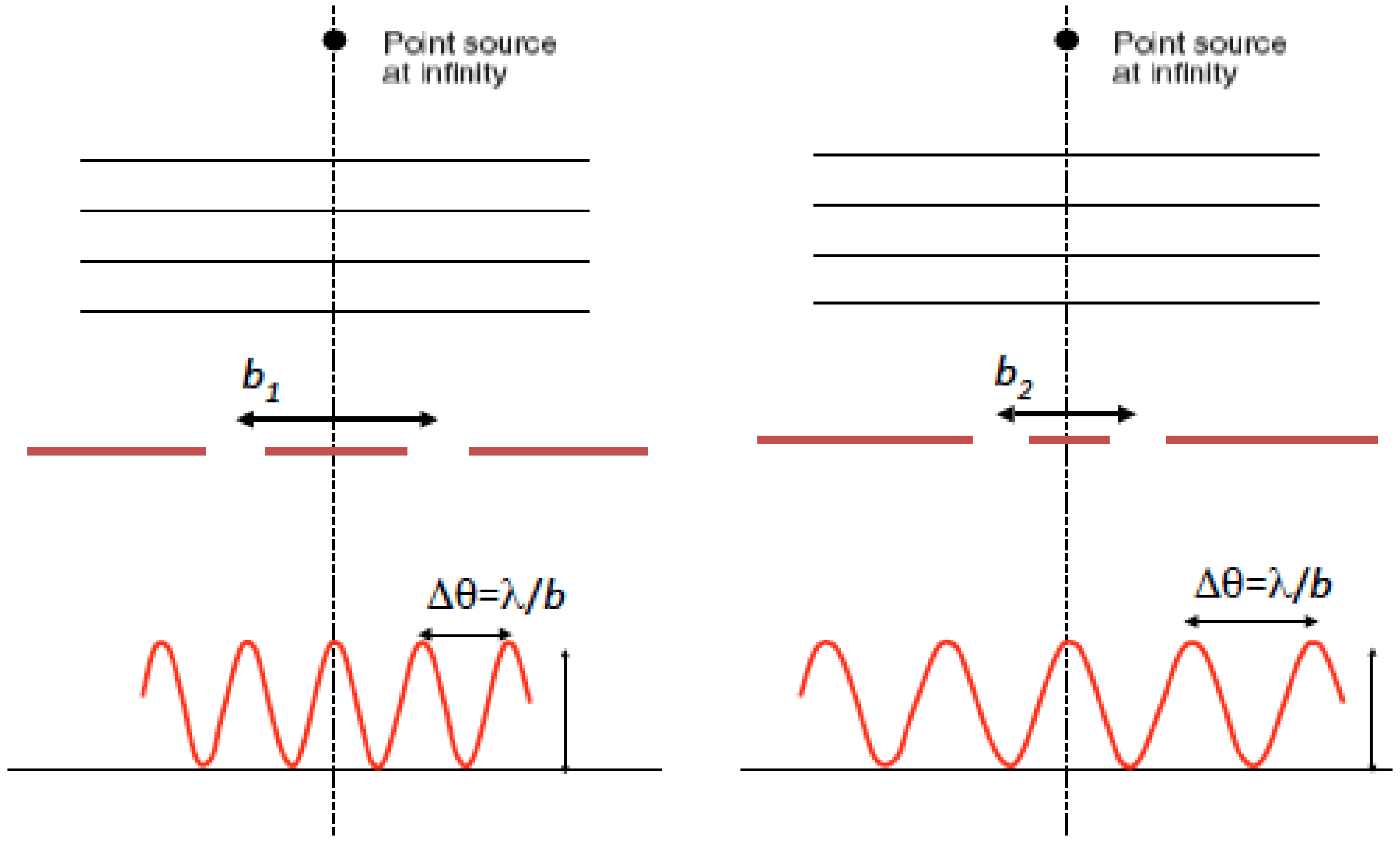
The size of the spot is determined by the **diameter, D,** of the aperture, and **wavelength, λ,** of the incident light.

Diffraction by a circular aperture is similar to single-slit diffraction. But note the difference:

Slit	$\theta_0 \approx \dfrac{\lambda}{a}$
Circular aperture	$\theta_0 \approx 1.22\,\dfrac{\lambda}{D}$

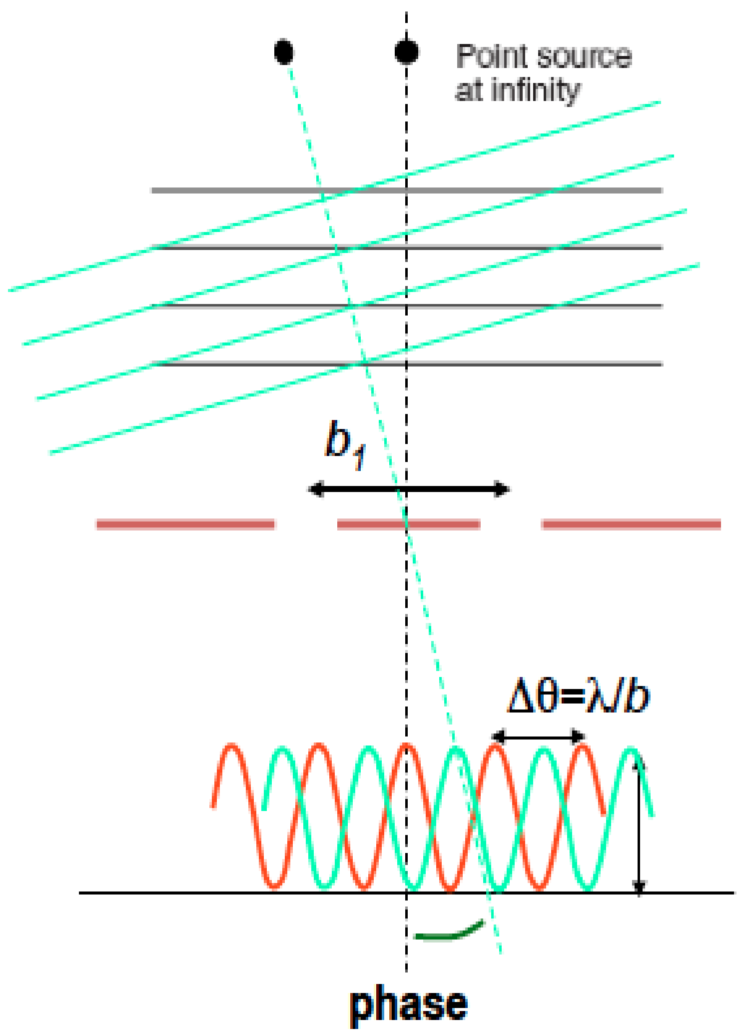
!!

Interferometers: the basics



Spatial frequency : ʎ/b

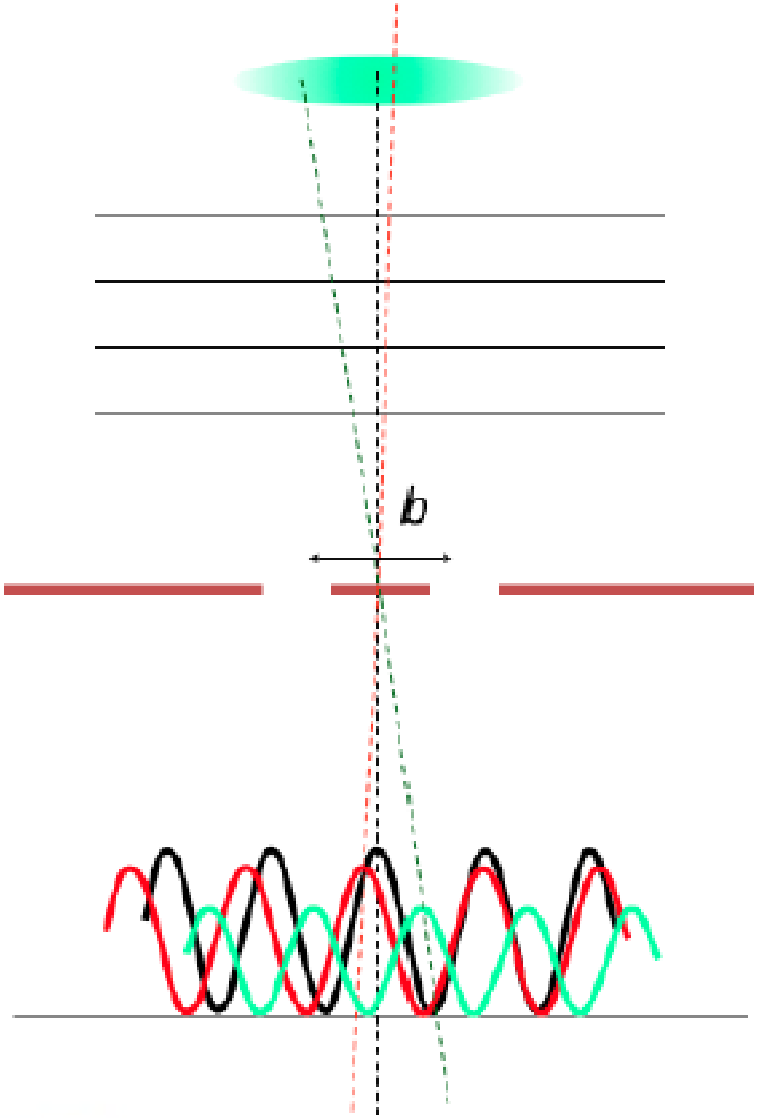
Interferometers: the basics



Spatial Frequency λ\b

if we observe at different λ, we sample different spatial frequencies

Interferometers: the basics



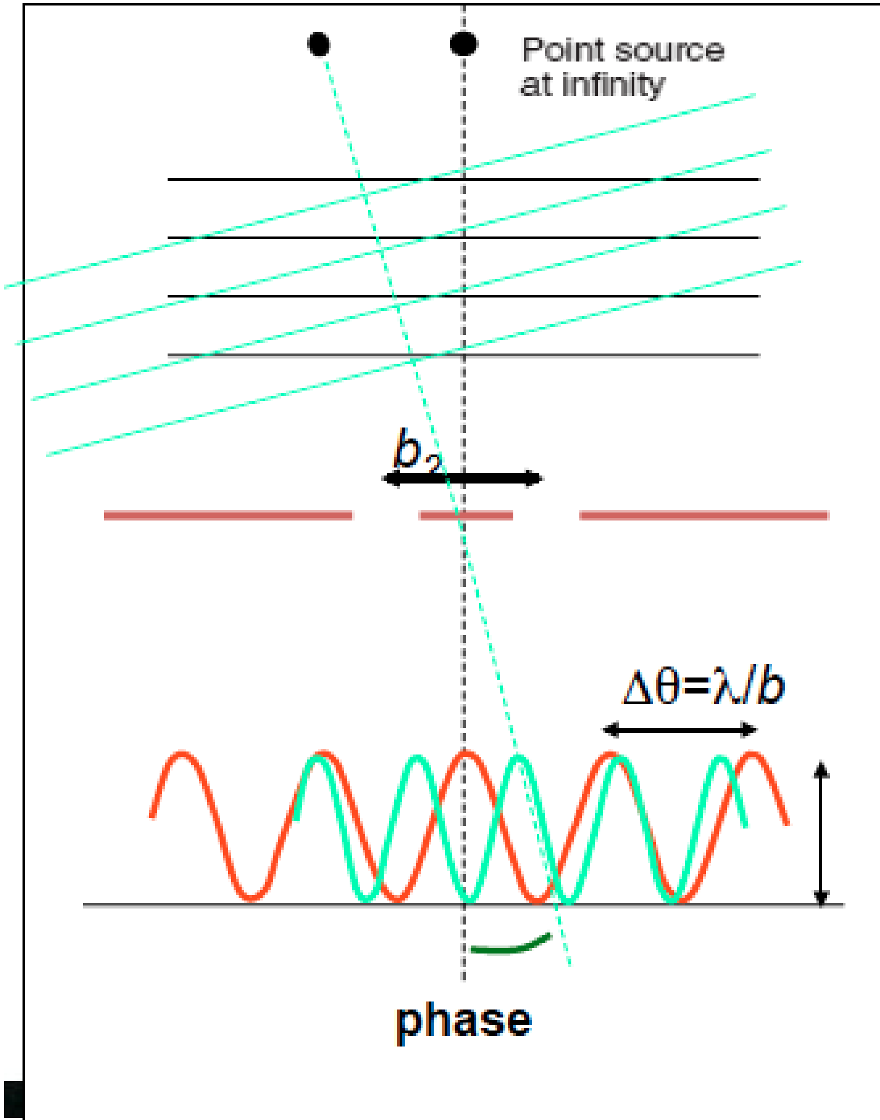
- Amplitude tells "how much" of a certain frequency component
- Phase tells "where" this component is located

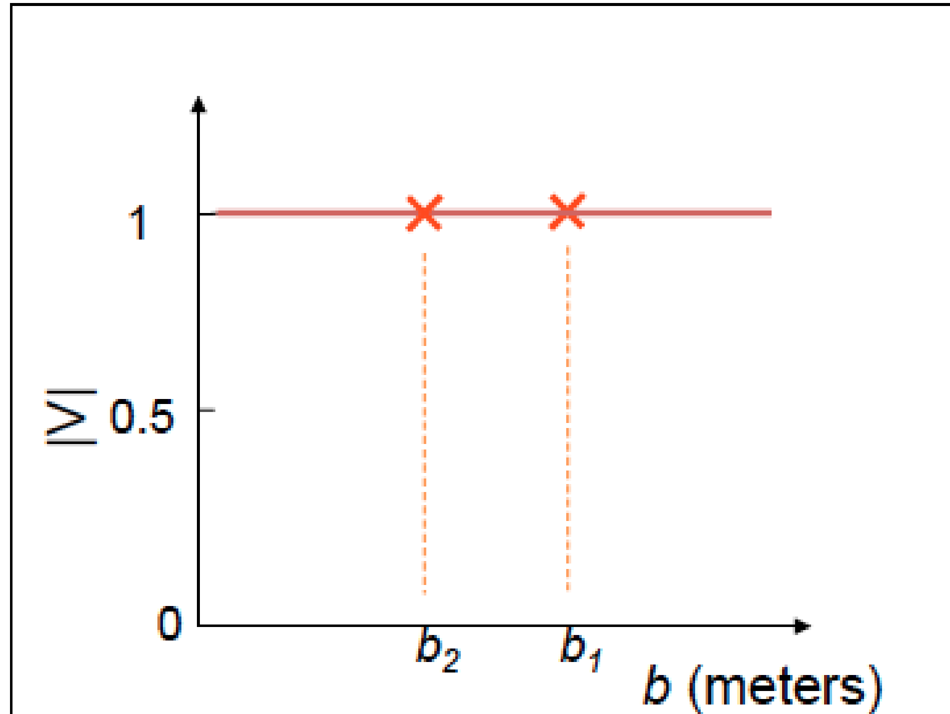
⬇

Visibility

Visibility and Sky Brightness

Graphic courtesy Andrea Isella





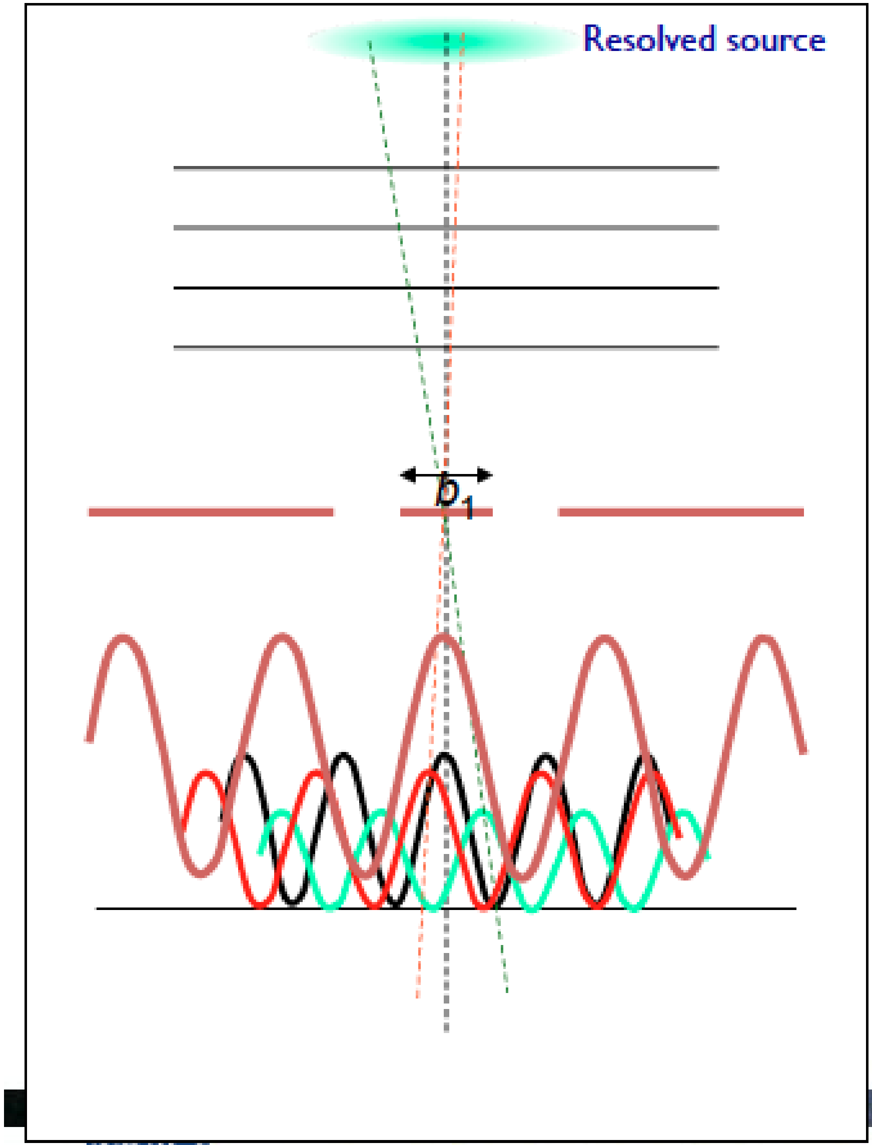
$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\textit{Fringe Amplitude}}{\textit{Average Intensity}}$$

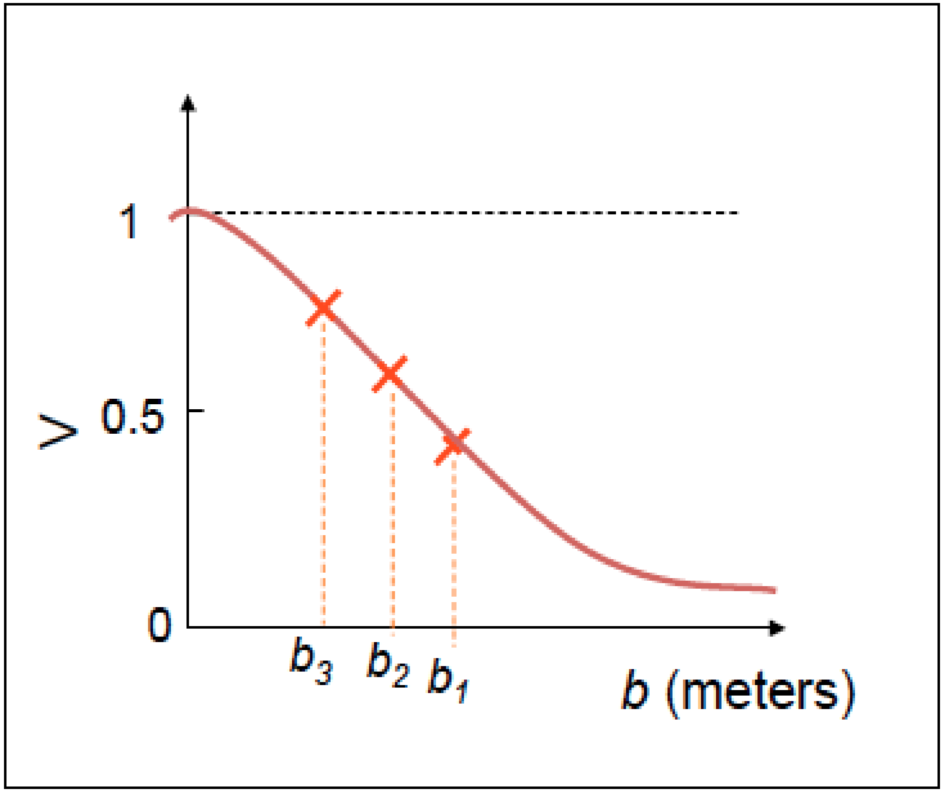
The visibility is a complex quantity:

- amplitude tells "how much" of a certain frequency component
- phase tells "where" this component is located

Visibility and Sky Brightness

Graphic courtesy Andrea Isella

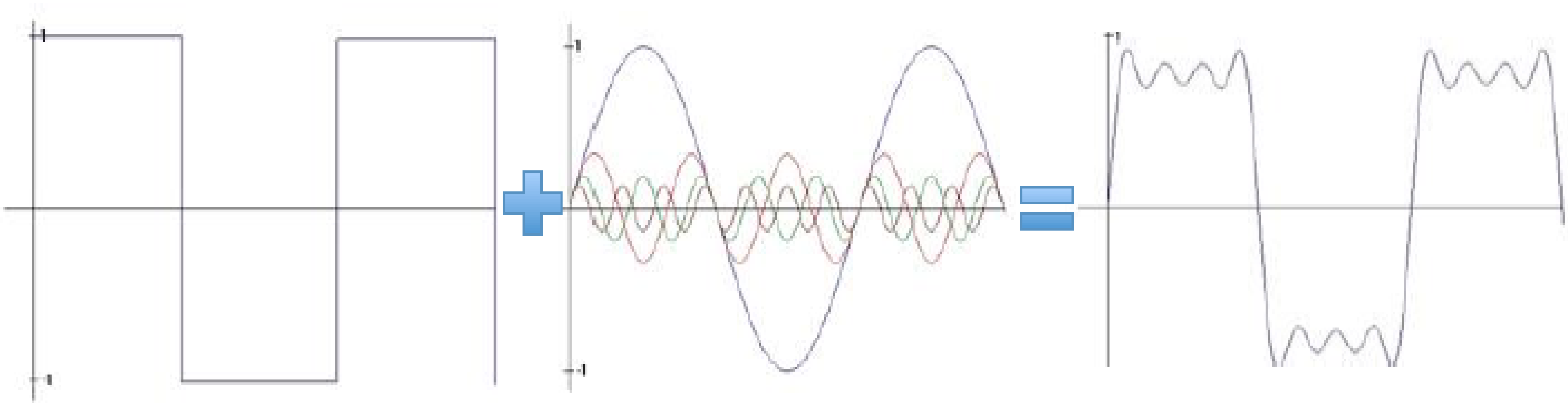




$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

The Fourier Transform

Fourier theory states that any well behaved signal (including images) can be expressed as the sum of sinusoids



Reference signal 4 sinusoids Sum of sinusoids & signal

- The Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components
- The Fourier transform contains *all* of the information of the original signal

The Fourier Transform relates the measured interference pattern to the radio intensity on the sky

1. An interferometer measures the interference pattern produced by pairs of apertures.

2. The interference pattern is directly related to the source brightness:
   - For small fields-of-view: the complex visibility, V(u,v), is the 2D Fourier transform of the brightness on the sky, T(x,y)

(van Cittert-Zernike theorem)

The Fourier Transform relates the measured interference pattern to the radio intensity on the sky

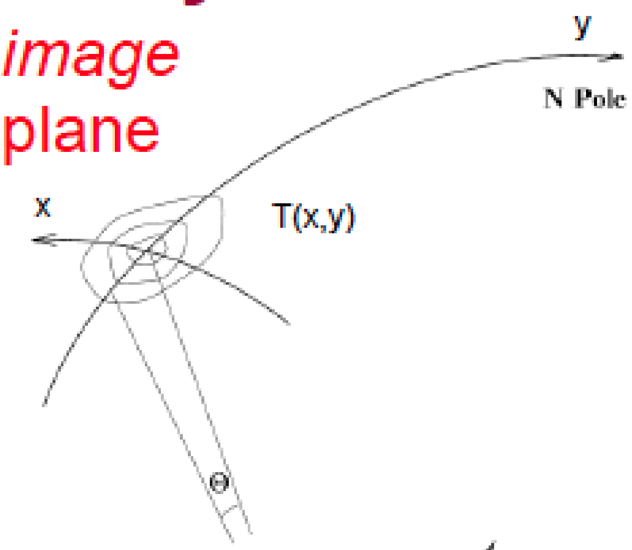
Fourier space/domain

$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dx dy$$

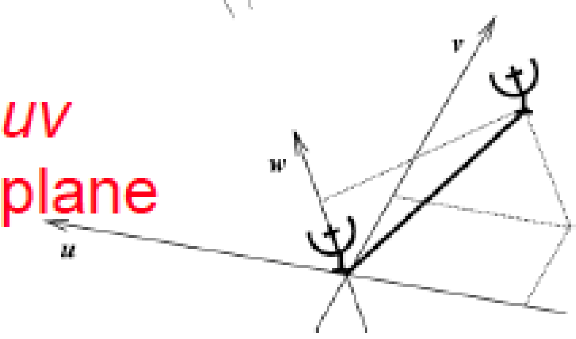
Image space/

$$T(x,y) = \int\int V(u,v) e^{-2\pi i(ux+vy)} du dv$$

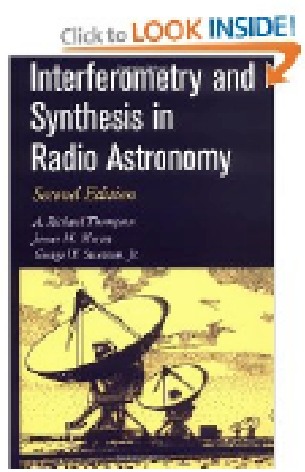
image plane



uv plane

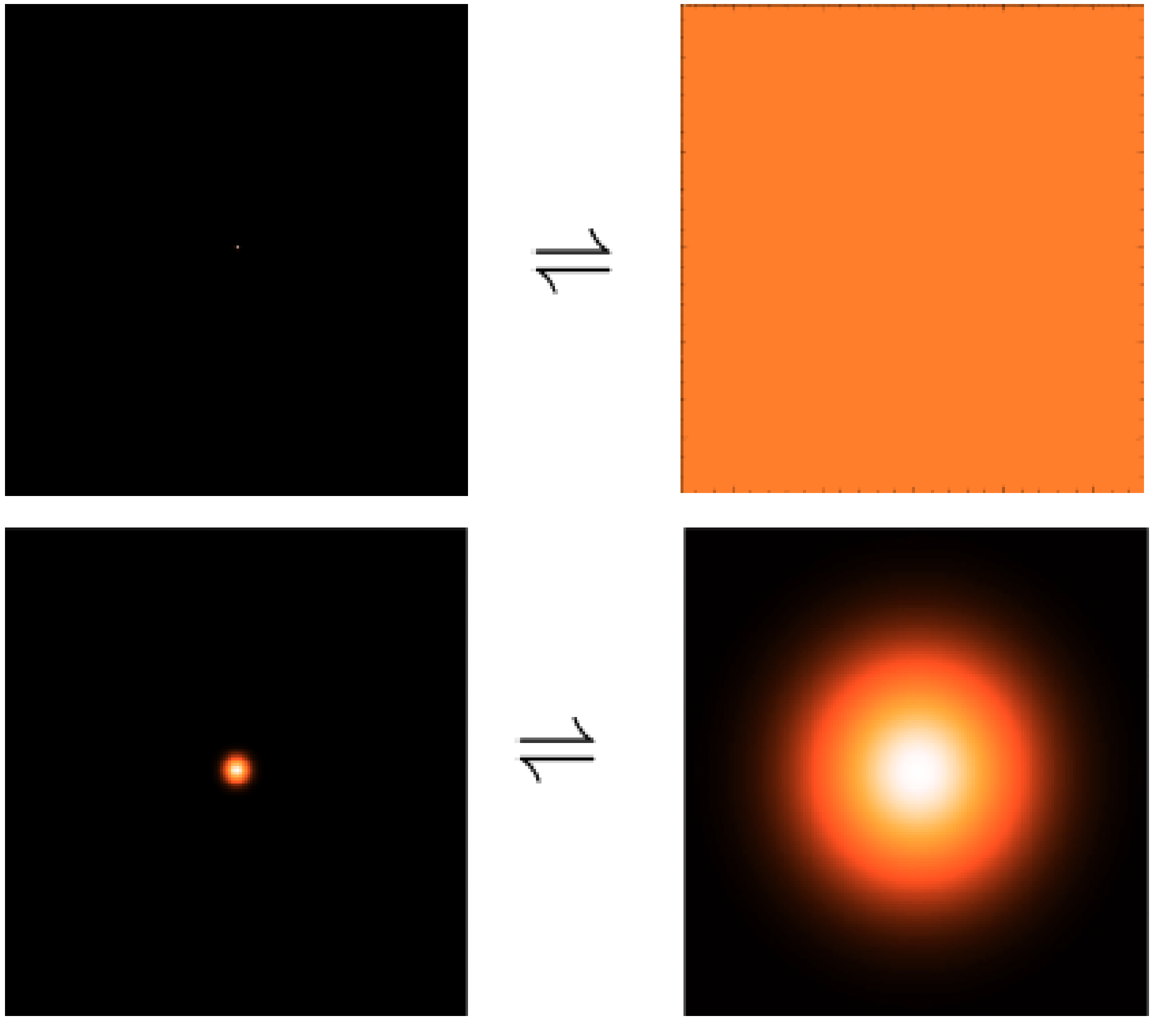


(for more info, see e.g. Thompson, Moran & Swenson)



Some 2D Fourier Transform Pairs

T(x,y) Amp{V(u,v)}



δ Function Constant

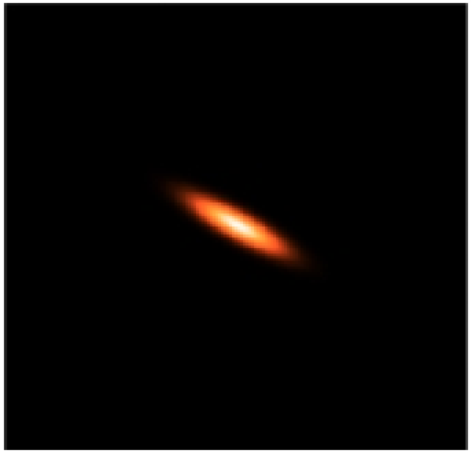
Gaussian Gaussian

narrow features transform to wide features (and vice-versa)

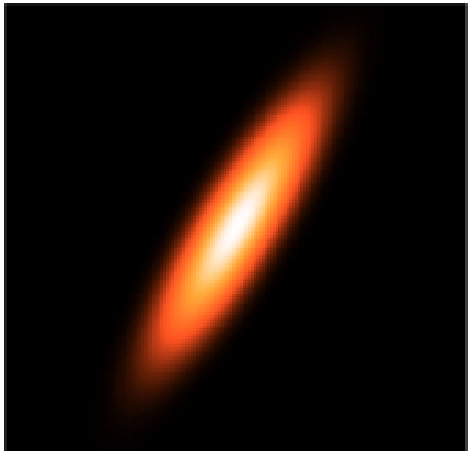
2D Fourier Transform Pairs

T(x,y) Amp{V(u,v)}

elliptical Gaussian

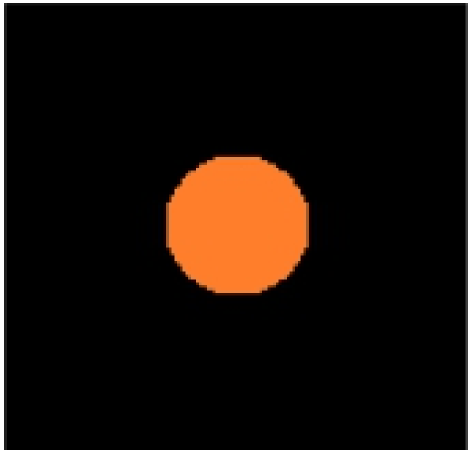


$\rightleftharpoons$

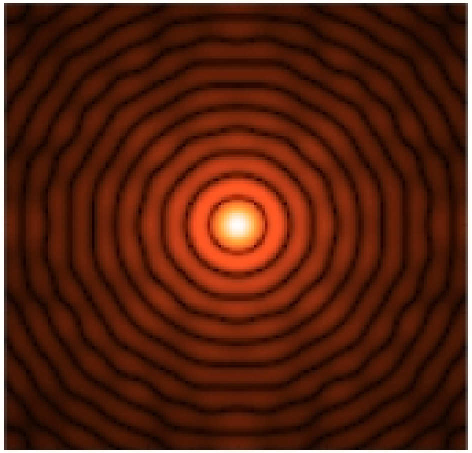


elliptical Gaussian

Disk



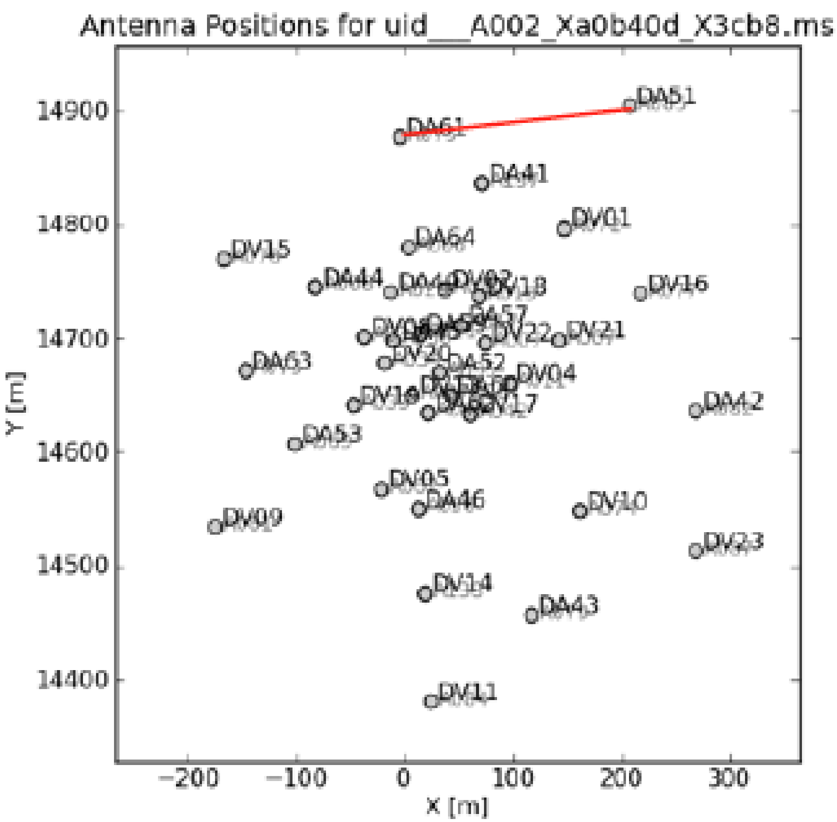
$\rightleftharpoons$



Bessel

Interferometers: the baseline in the uv plane

each antenna pair —> a point in uv plane

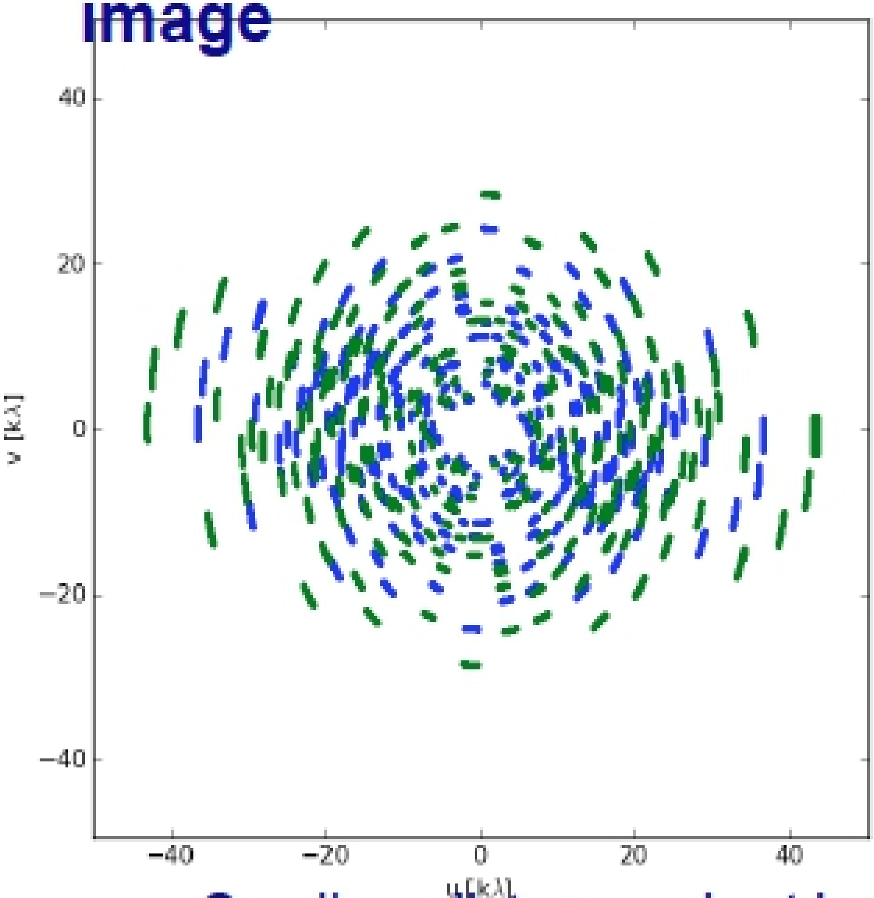


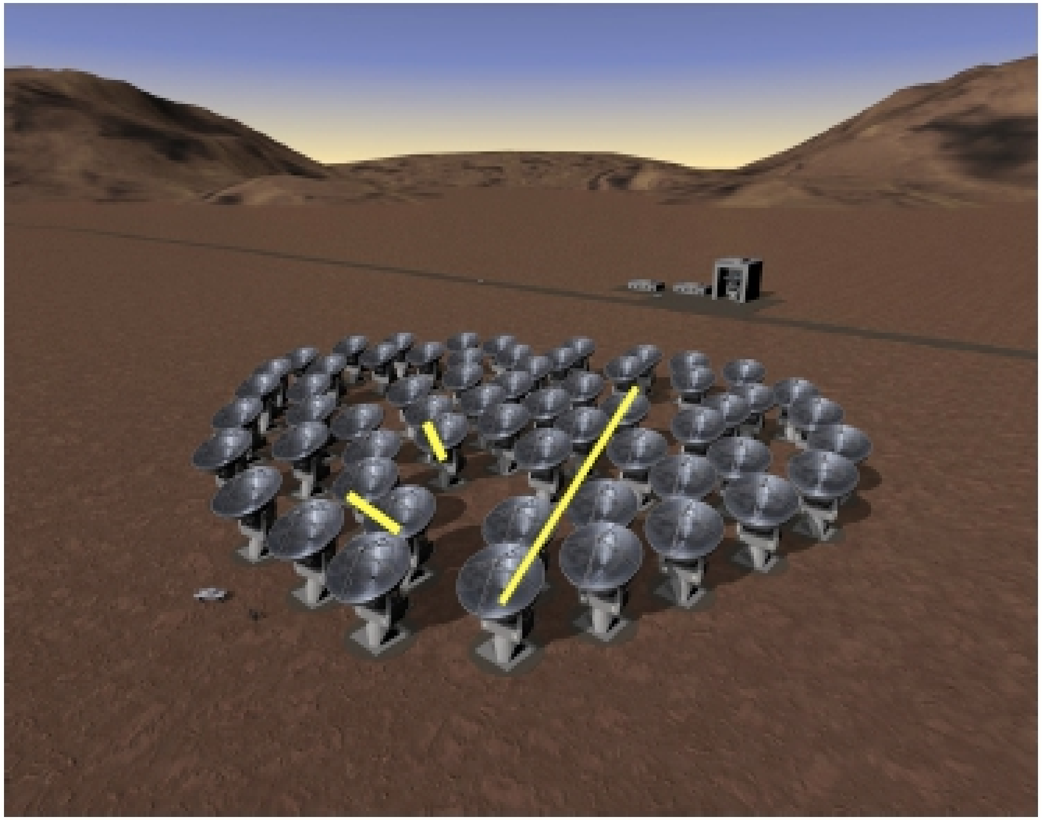
b(u,v) in λ units

u, v components in the E-W,N-S directions

Sampling Function

Each antenna pair samples only one spot; the array cannot sample the entire Fourier/uv domain resulting in an **imperfect image**





Small uv-distance: short baselines (measure extended emission)

Long uv-distance: long baselines (measure small scale emission)

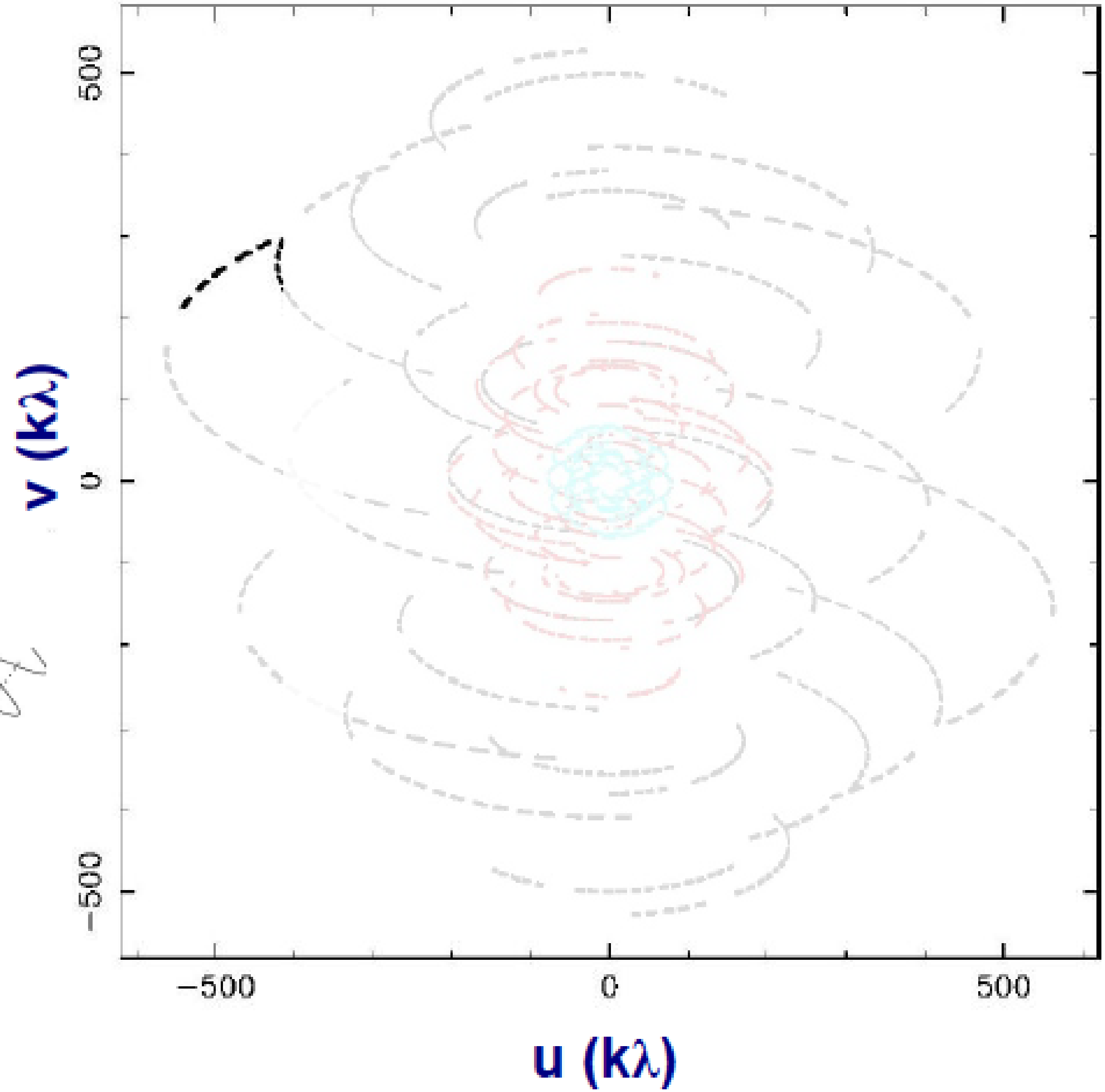
Orientation of baseline also determines orientation in the uv-plane

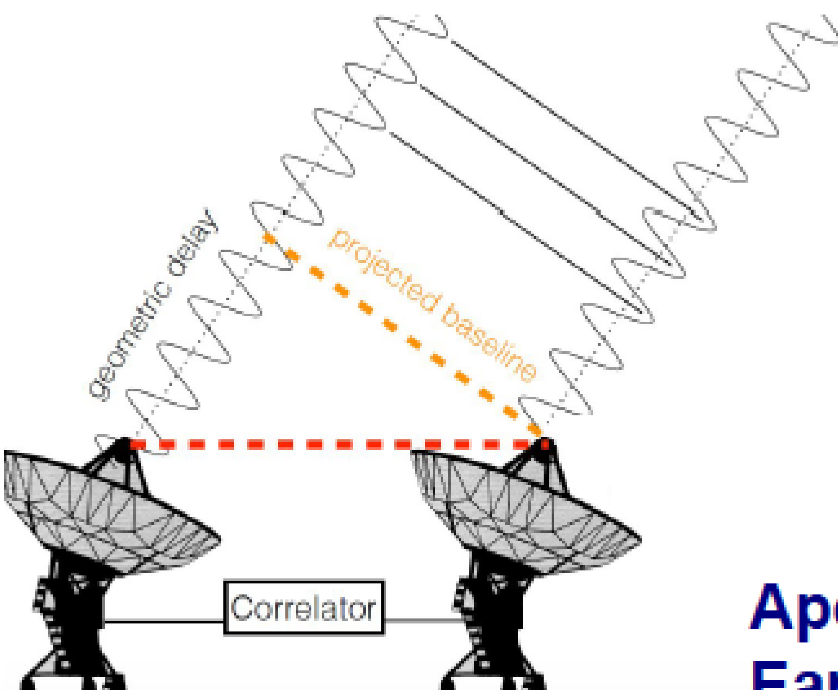
1 visibility

for each baseline

for each channel

for each t_{int} integration time

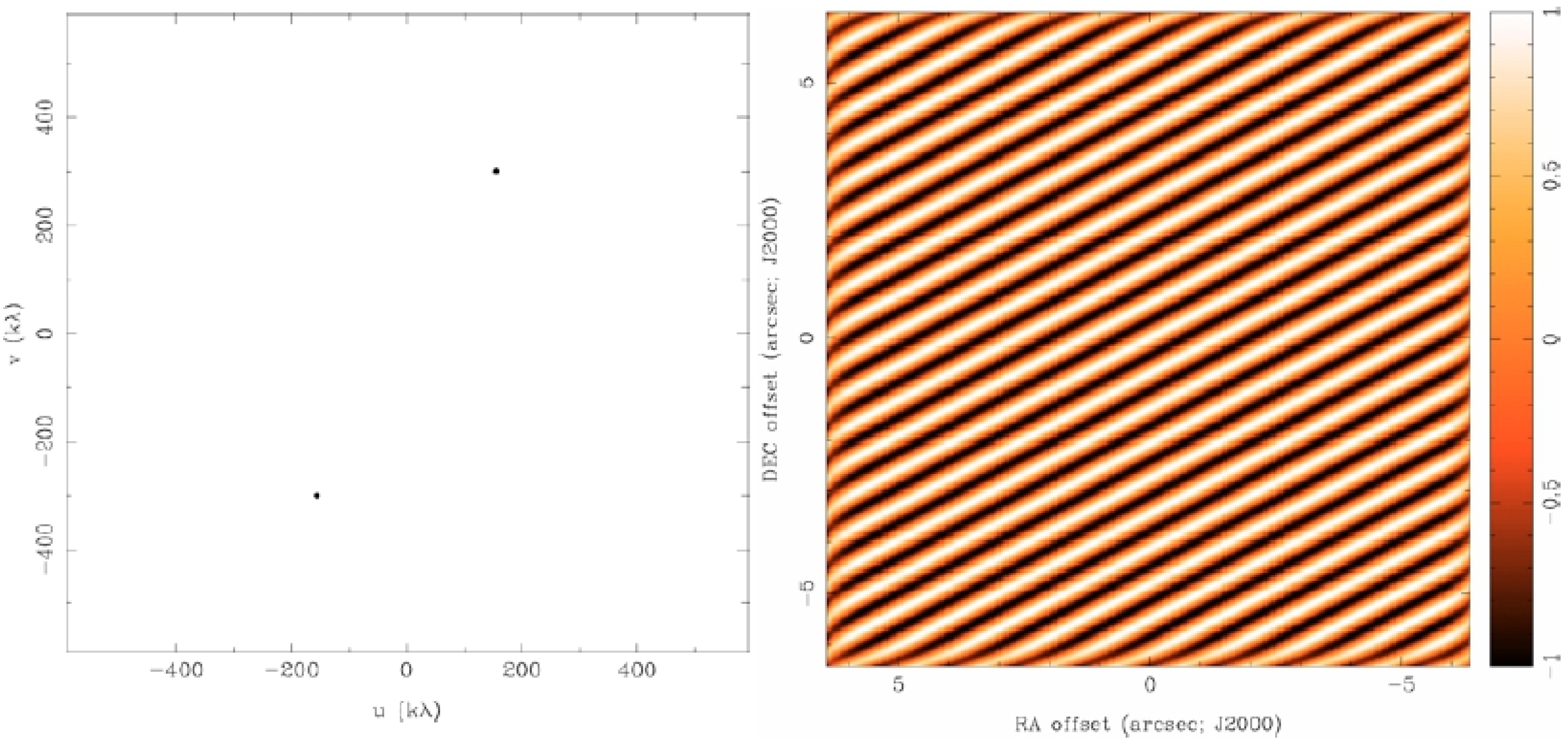




Aperture synthesis:
Earth rotation helps covering the uv plane

PSF shape vs. N ants

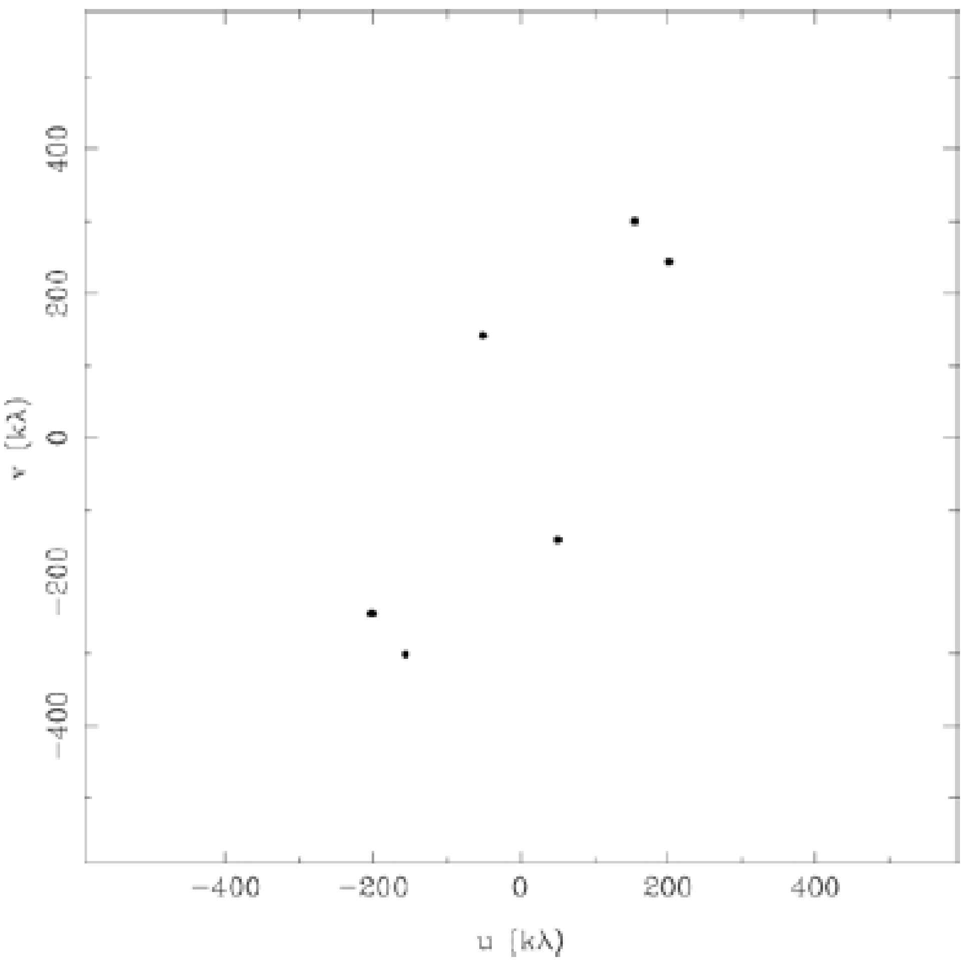
2 antennas

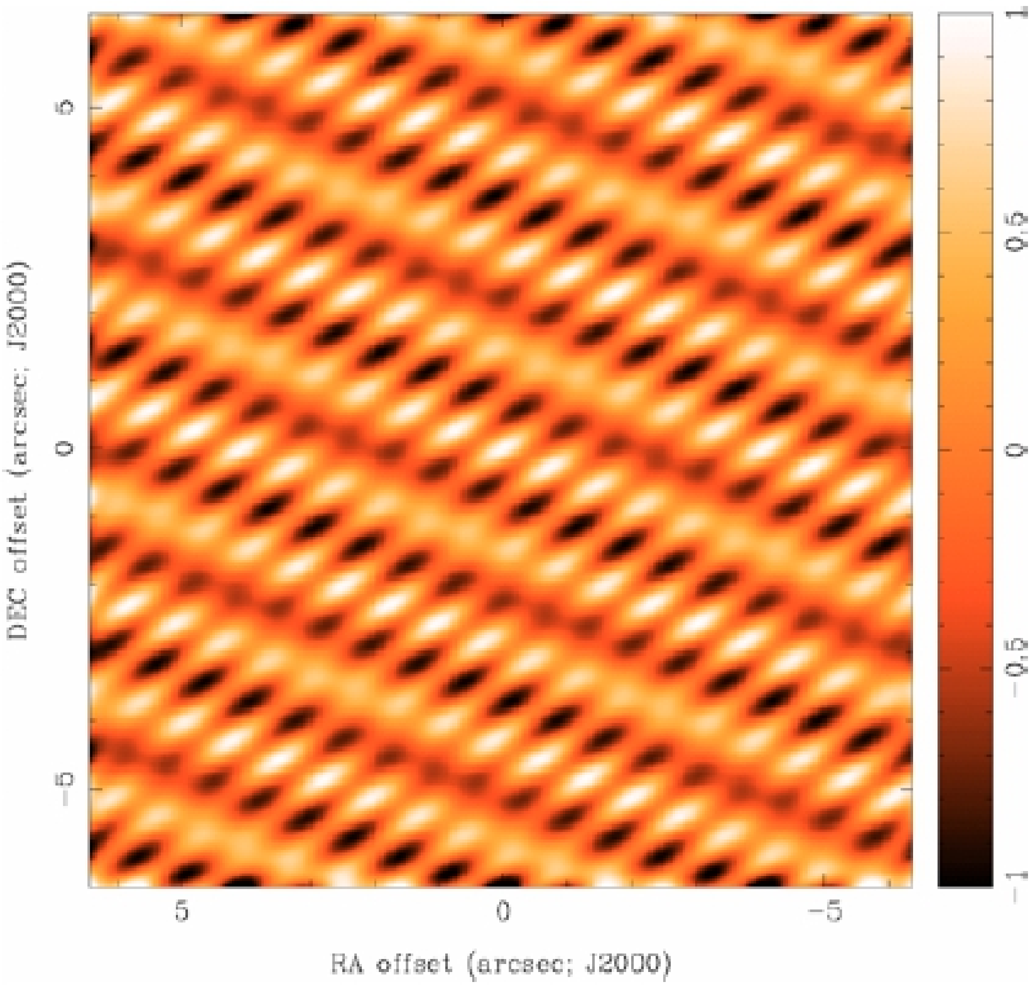


- to characterize a source, I need to sample as much as possible the uv plane.

PSF shape vs. N ants

3 antennas

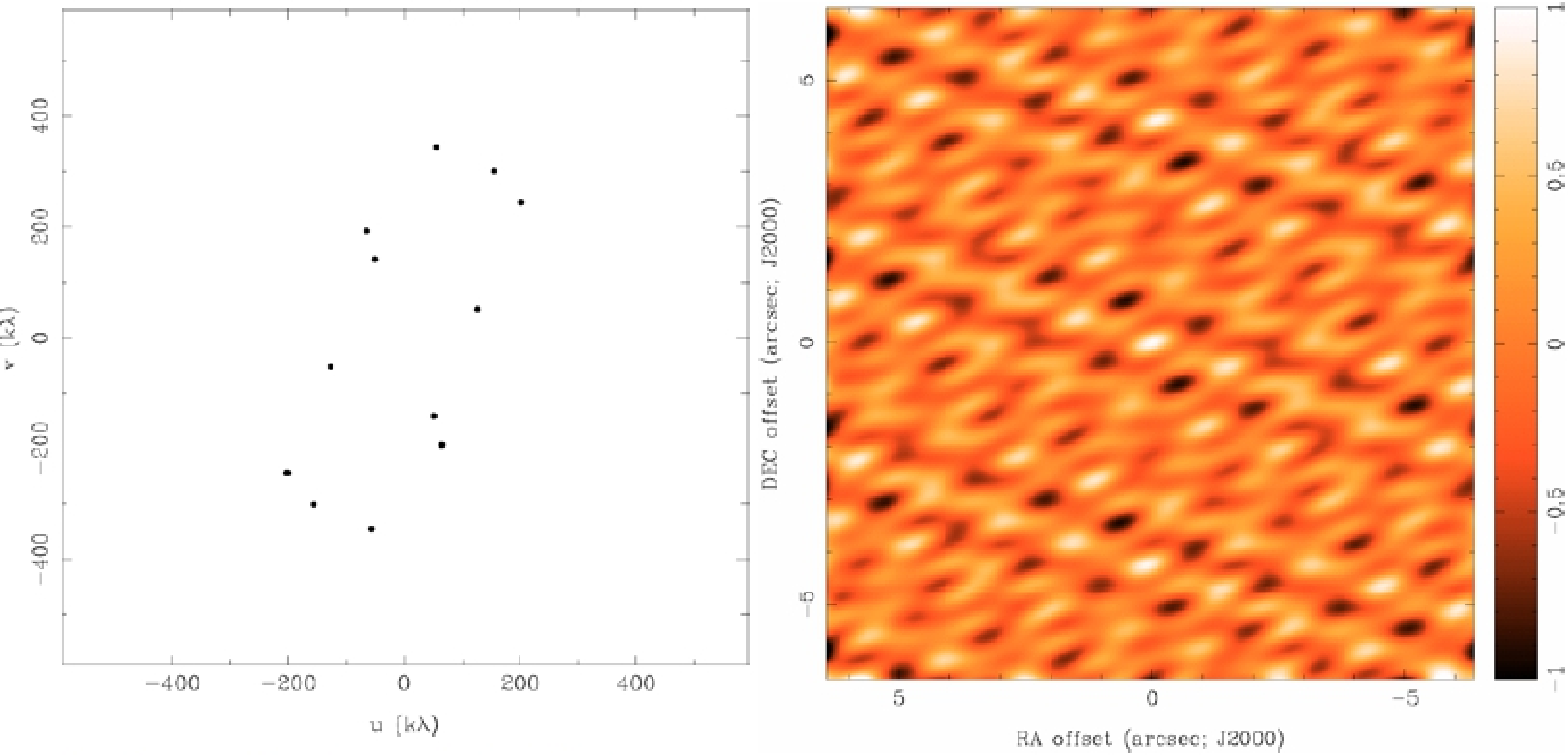




by increasing the number of antennas …

PSF shape vs. N ants

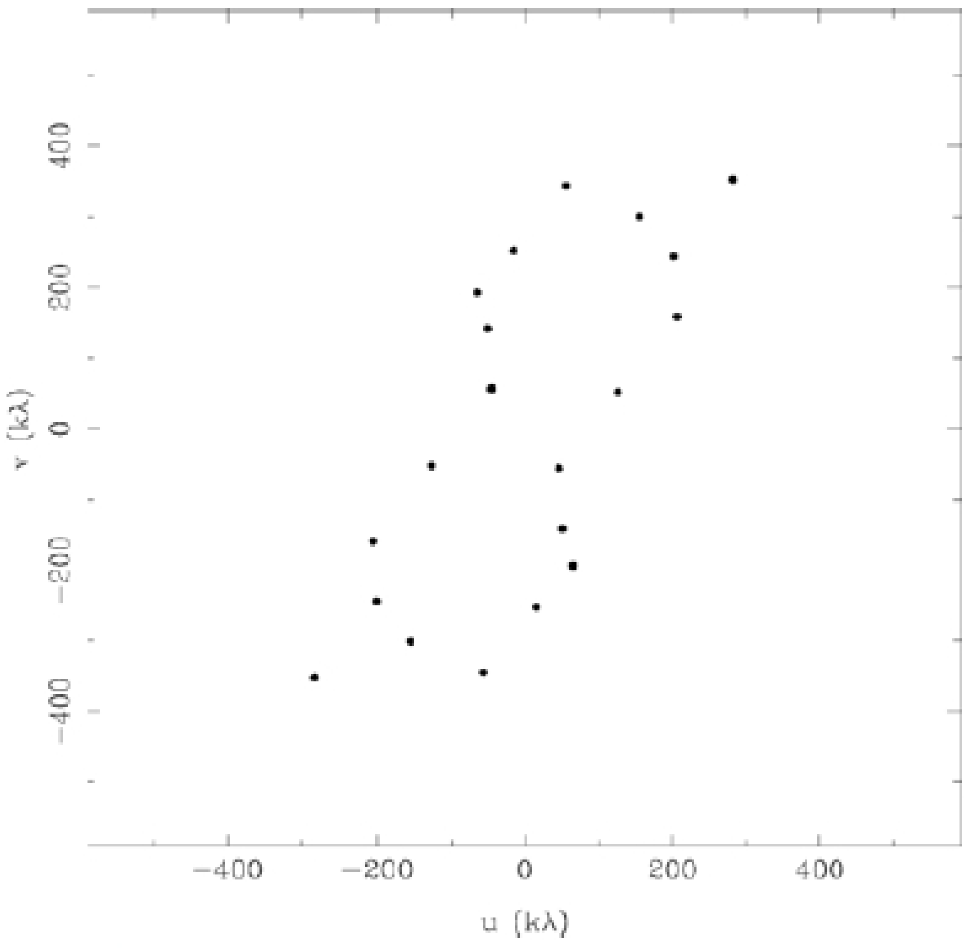
4 antennas

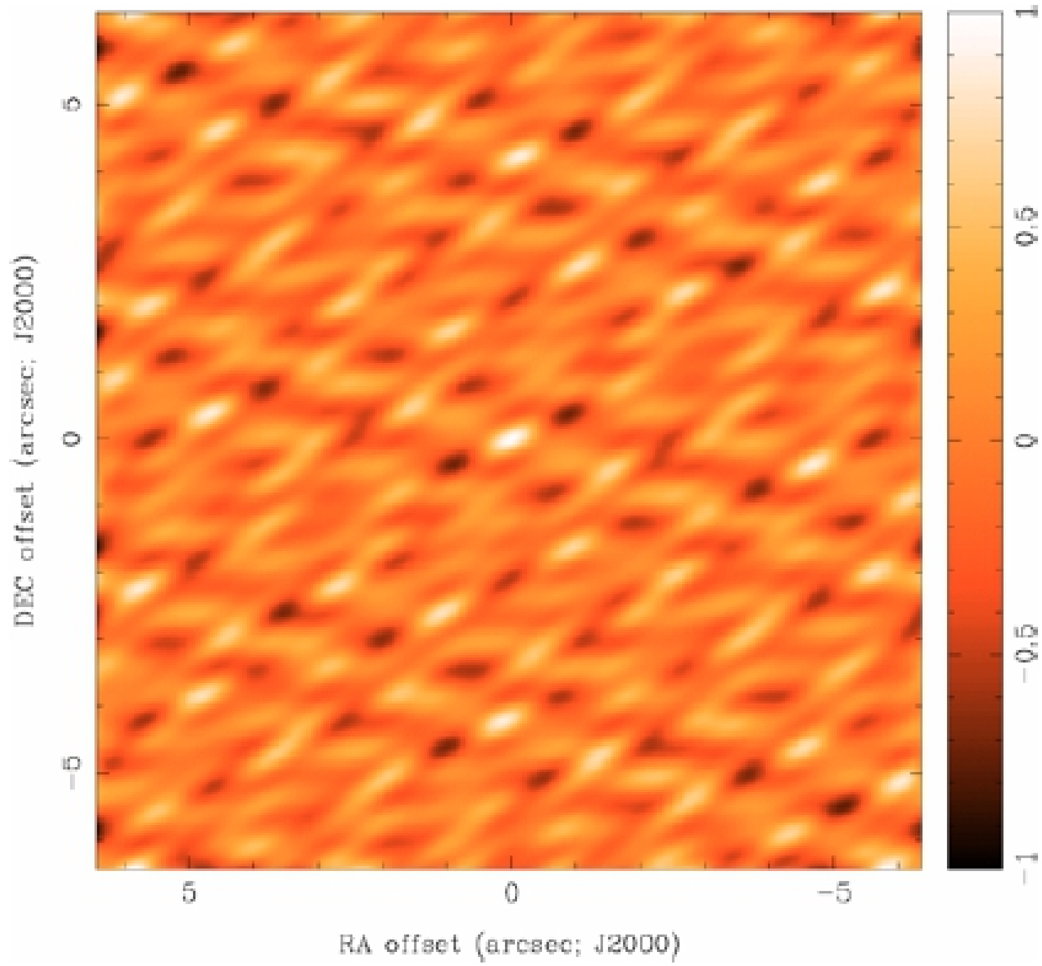


by increasing the number of antennas …

PSF shape vs. N ants

5 antennas

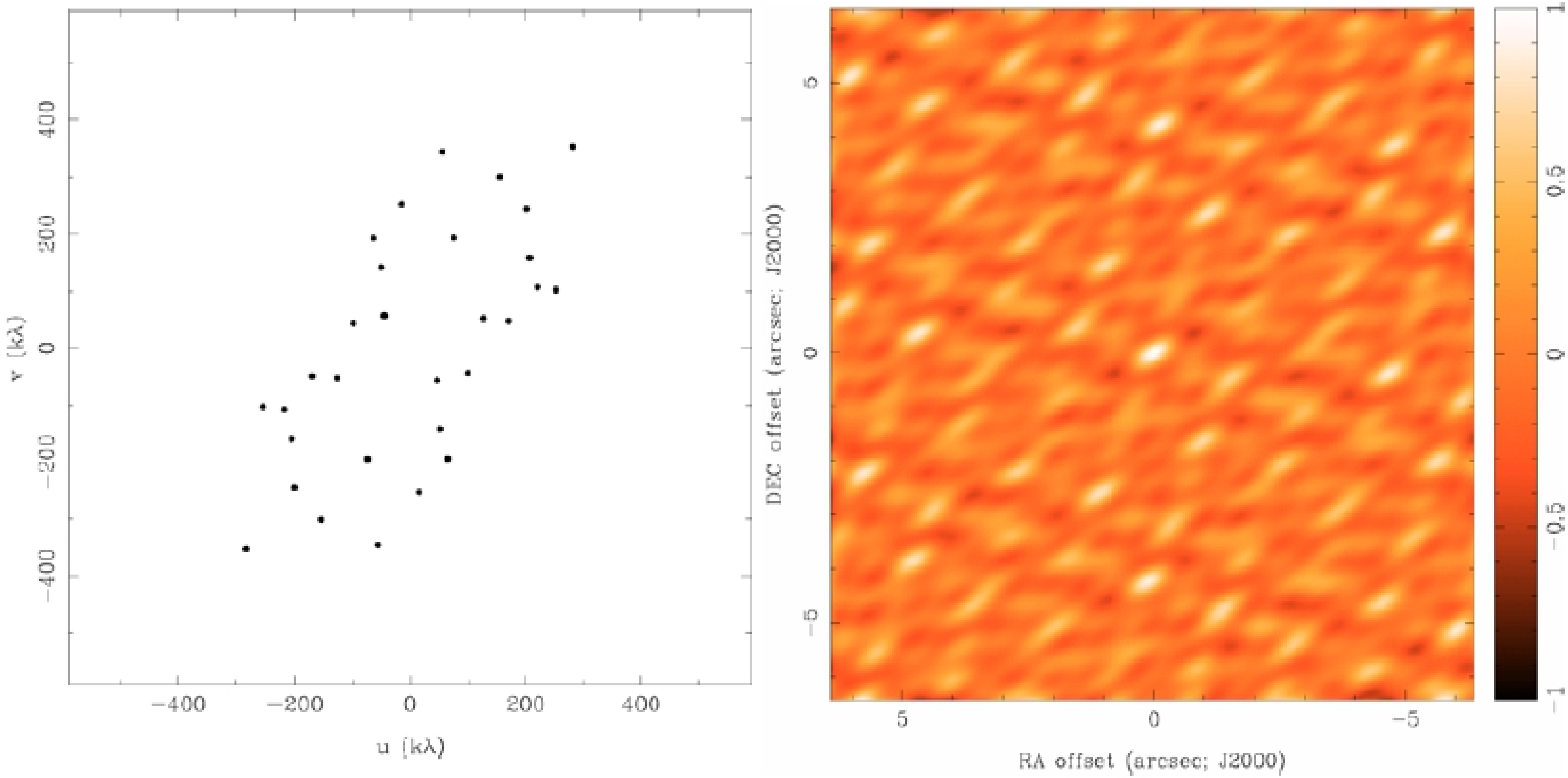




by increasing the number of antennas …

PSF shape vs. N ants

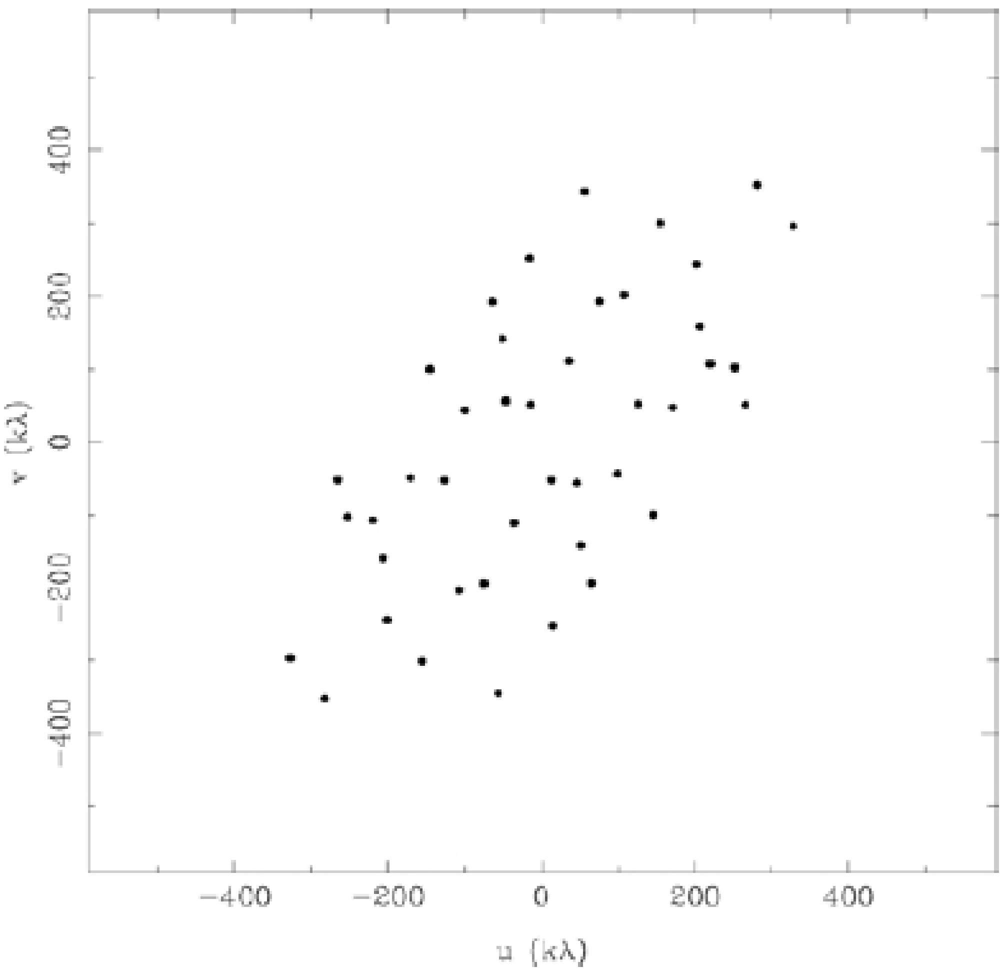
6 antennas

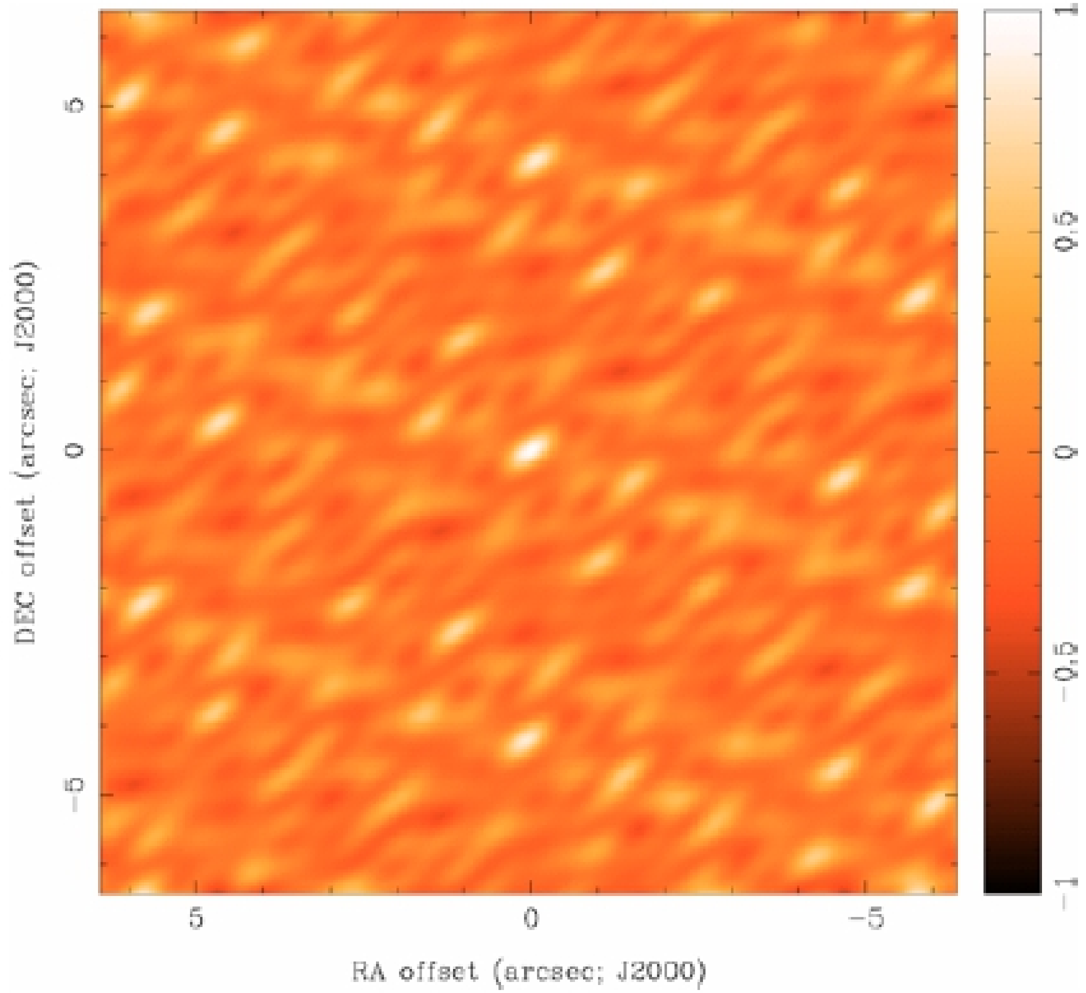


by increasing the number of antennas …

PSF shape vs. N ants

7 antennas

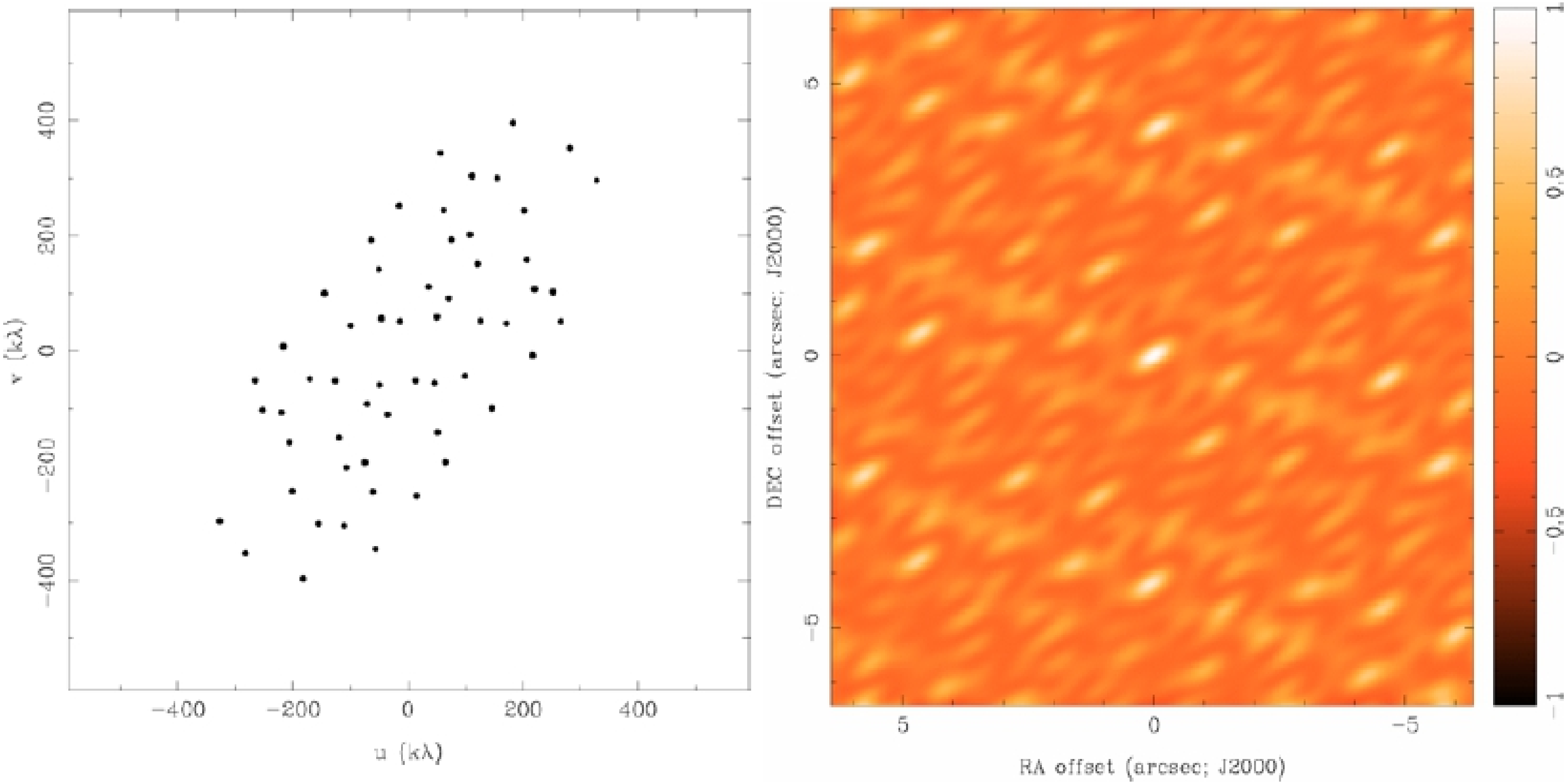




by increasing the number of antennas …

PSF shape vs. N ants

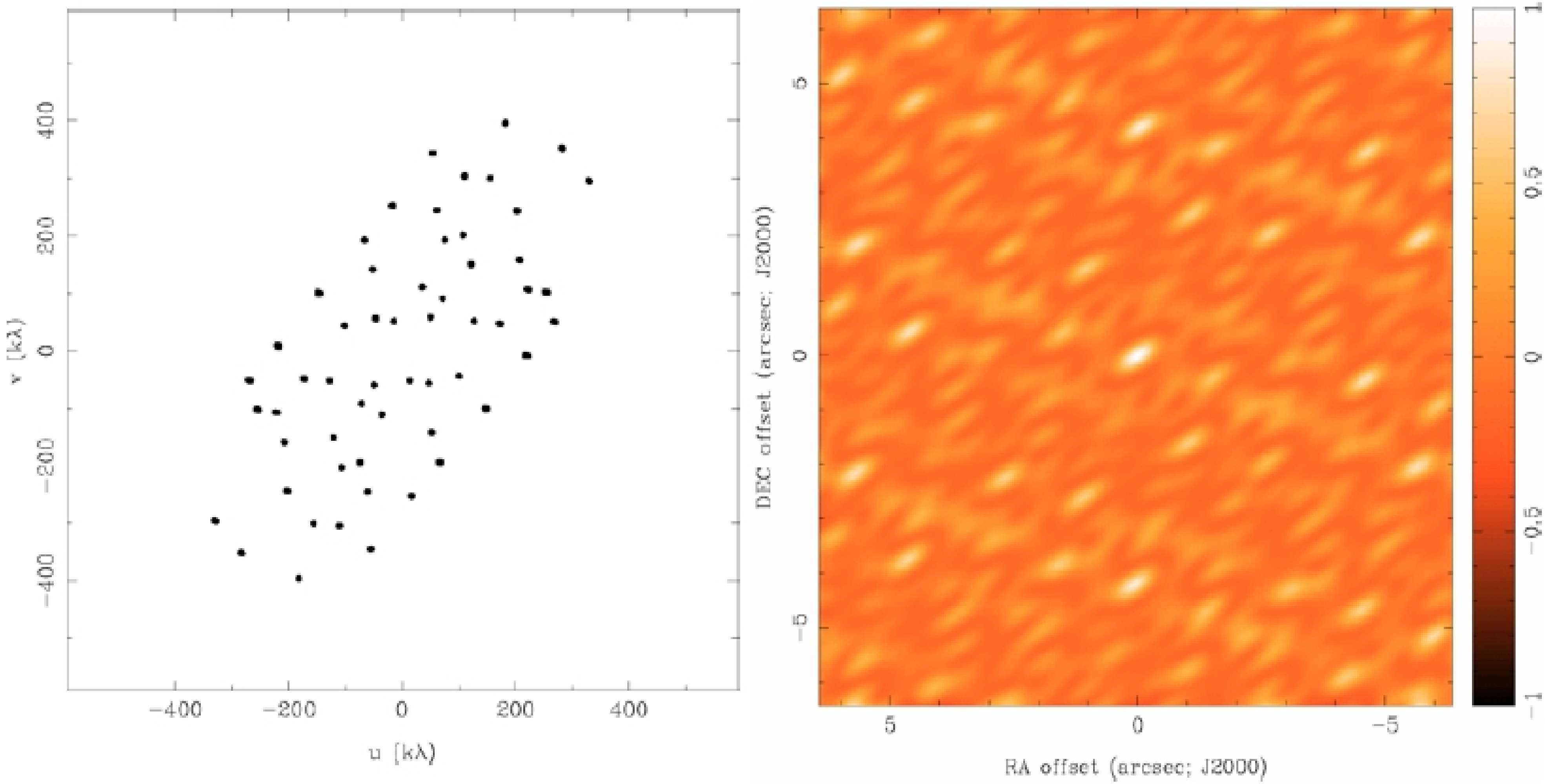
8 antennas



by increasing the number of antennas …

PSF shape vs. N ants

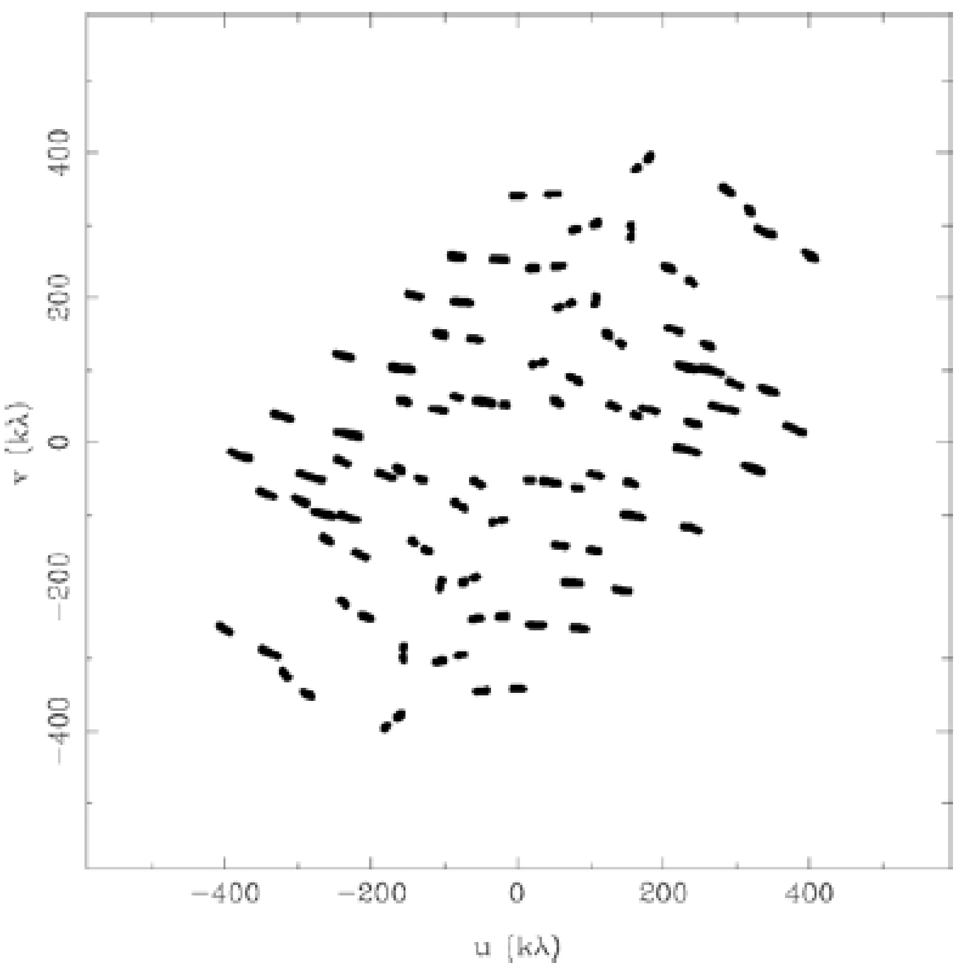
8 antennas x 6 samples

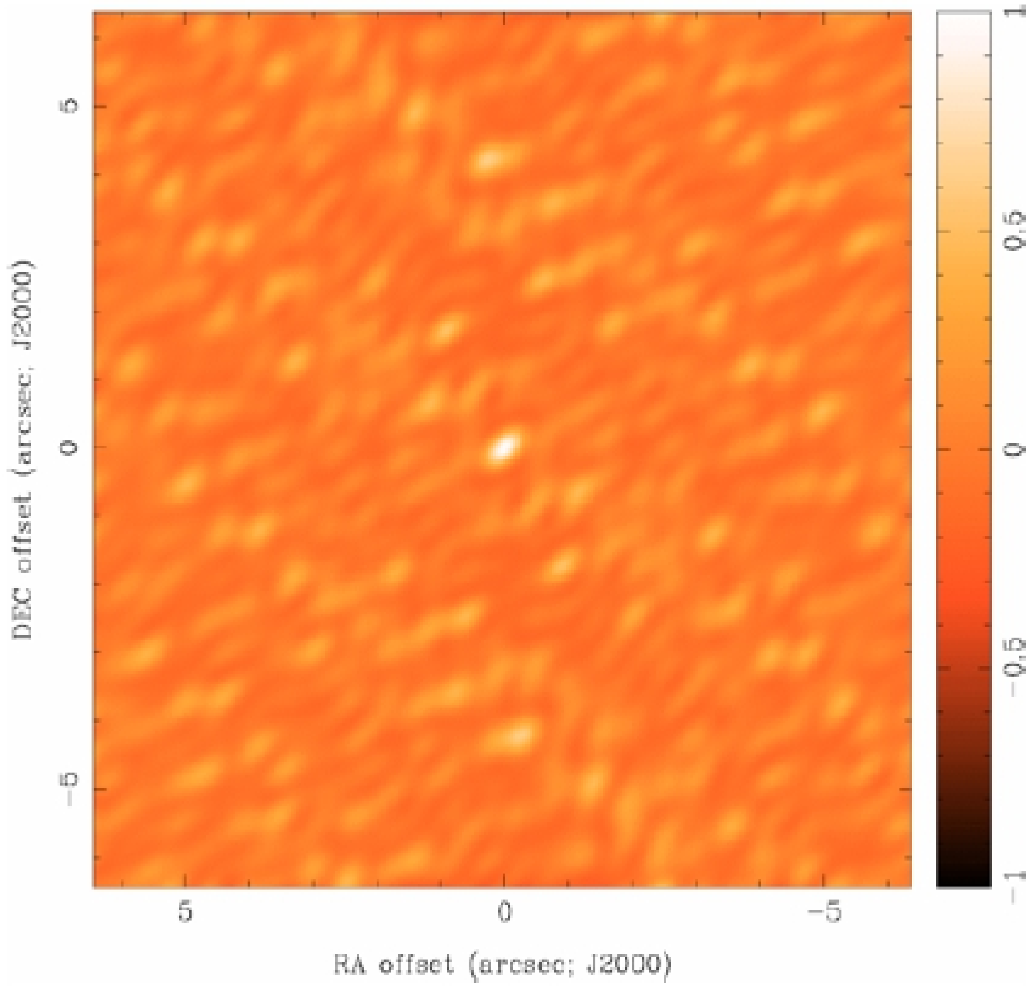


… or by increasing the integration time …

PSF shape vs. N ants

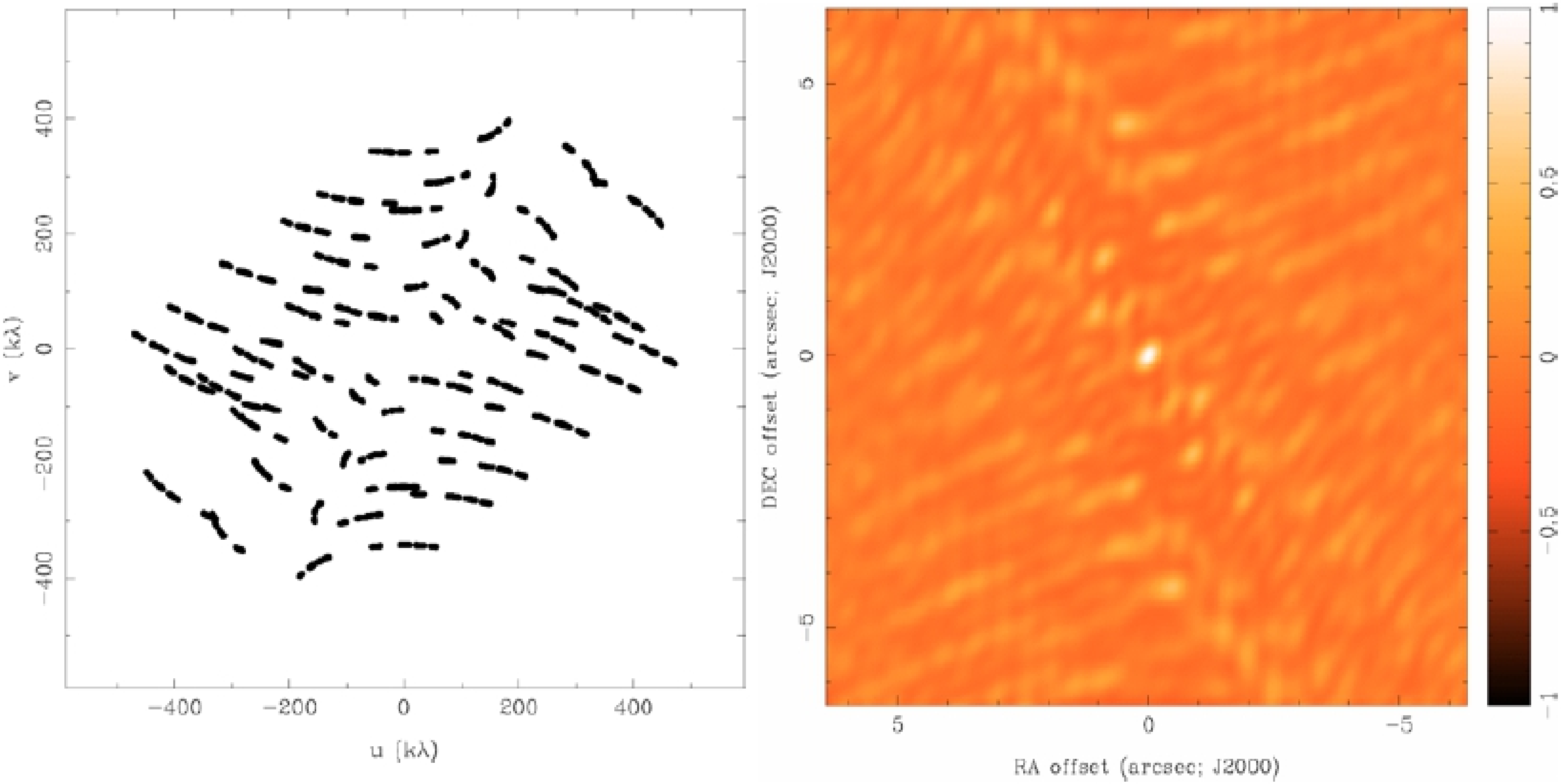
8 antennas x 60 samples





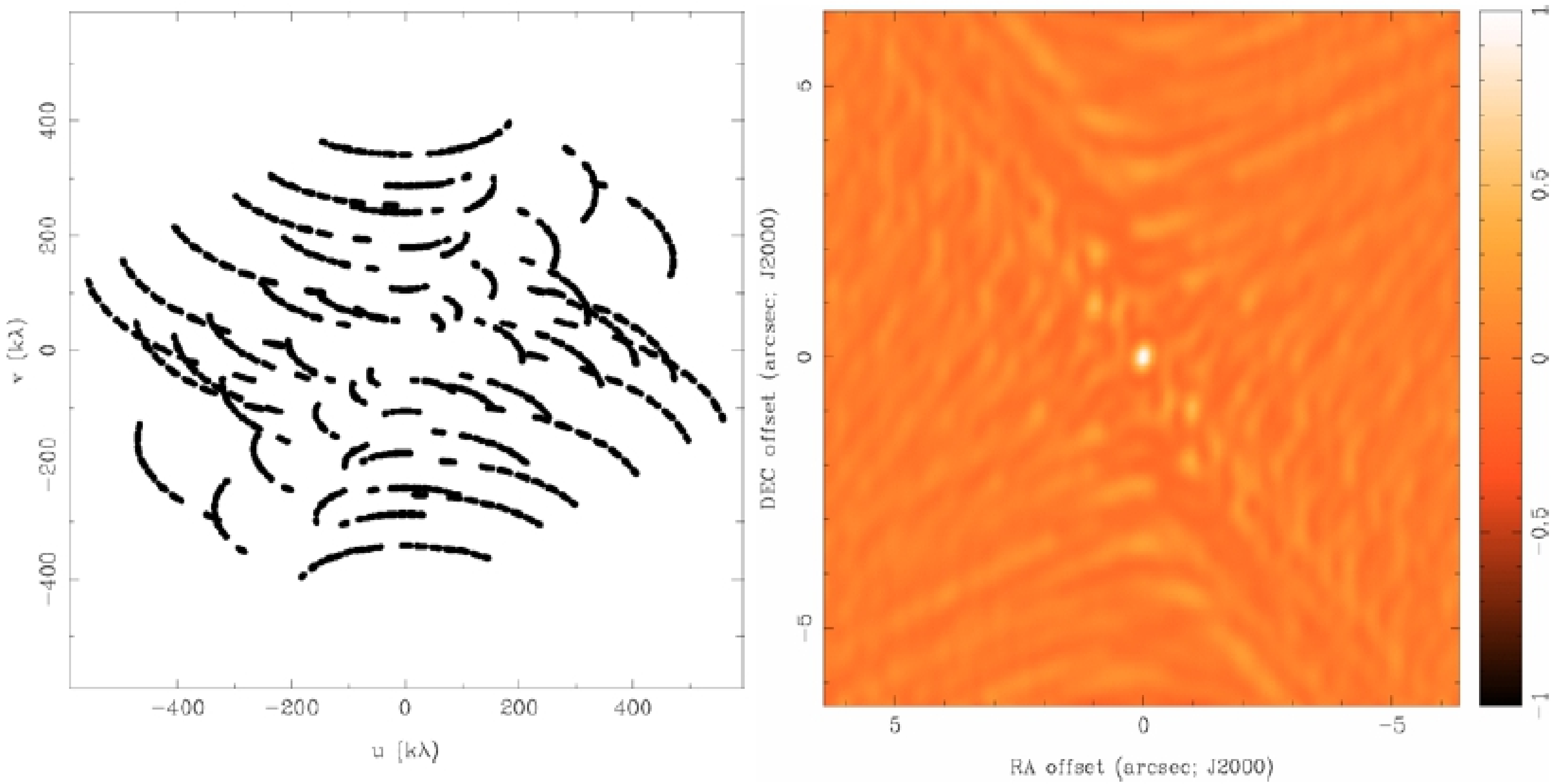
PSF shape vs. N ants

8 antennas x 120 samples



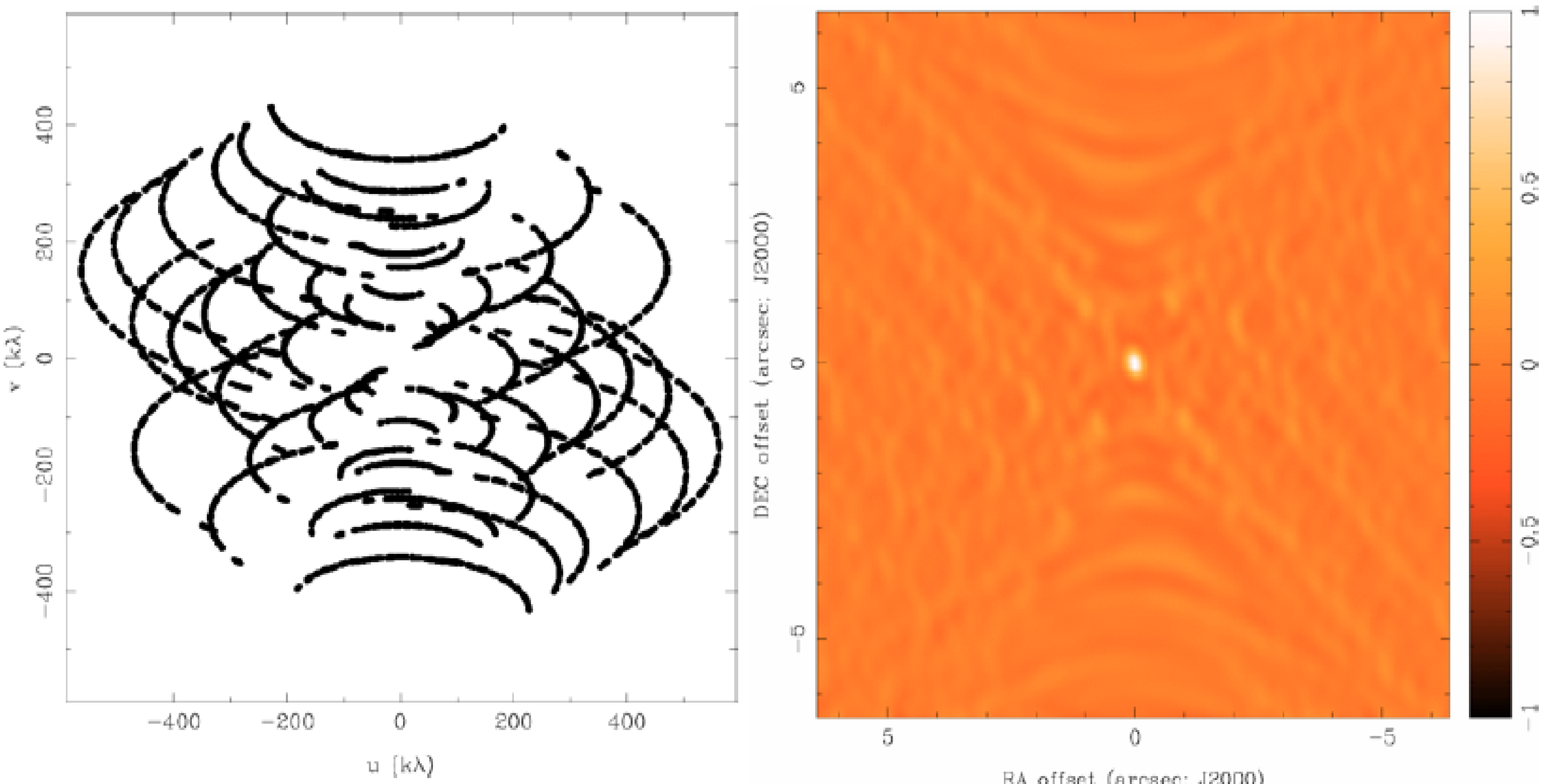
PSF shape vs. N ants

8 antennas x 240 samples



PSF shape vs. N ants

8 antennas x 240 samples



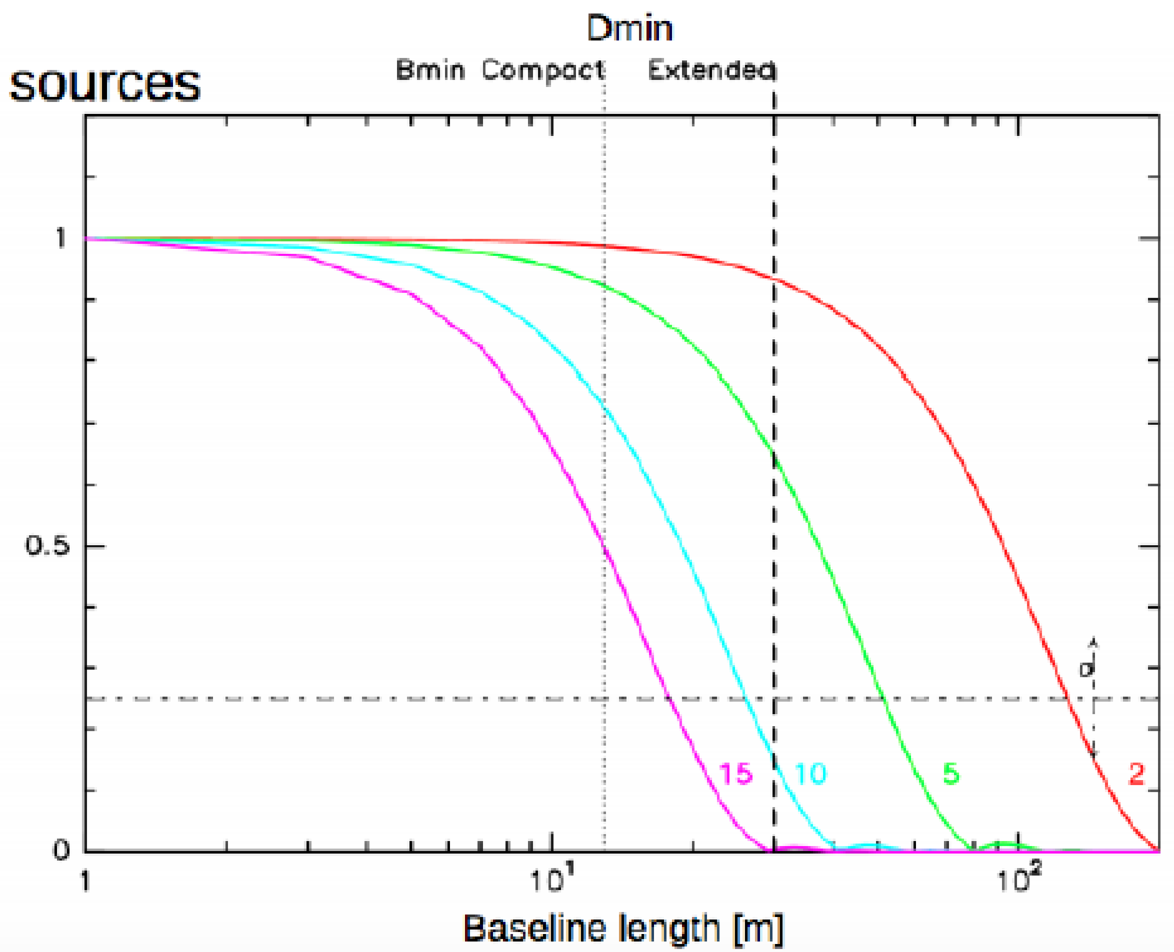
- ALMA has an *"instantaneous"* coverage uv plane…

sampling uv vs. MRS the zero spacing problem

Maximum recoverable scale

- Zero/ short spacing missing in interferometry
  - filtering of large scale emission

- Gaussian sources



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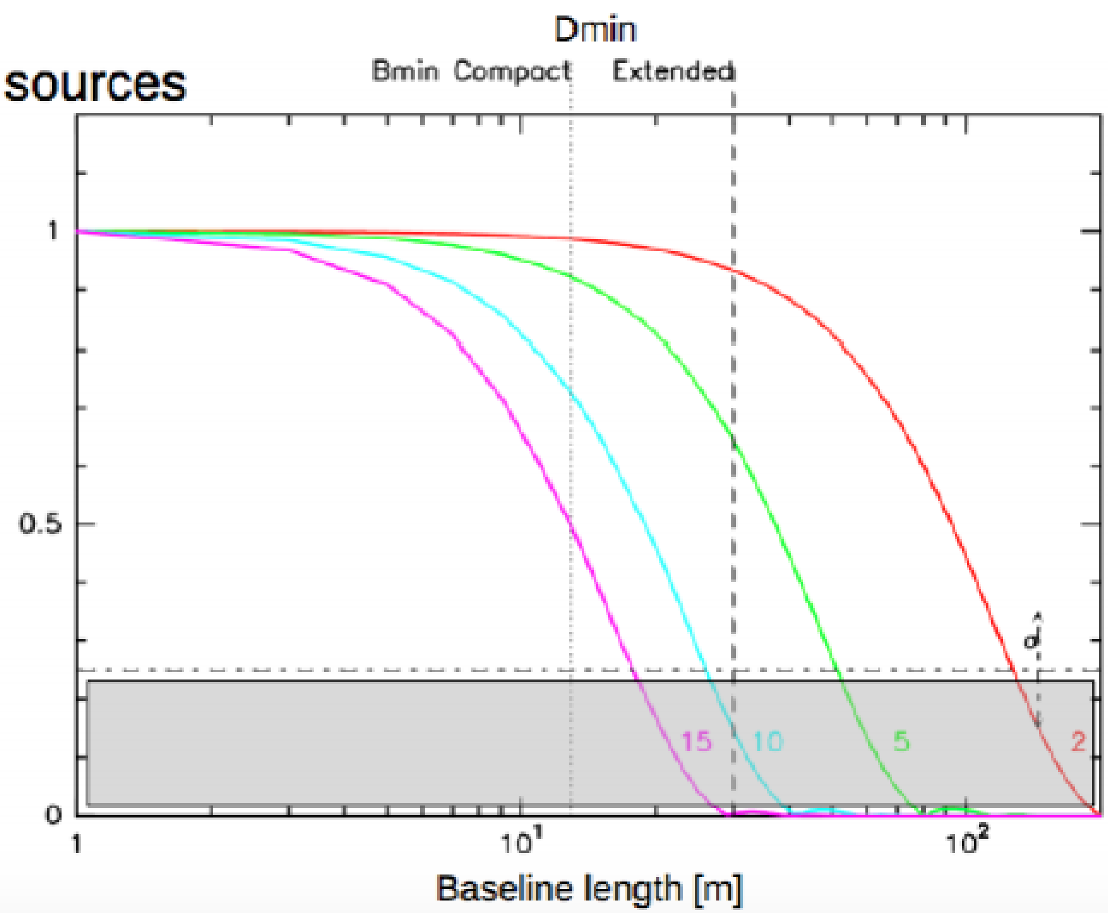

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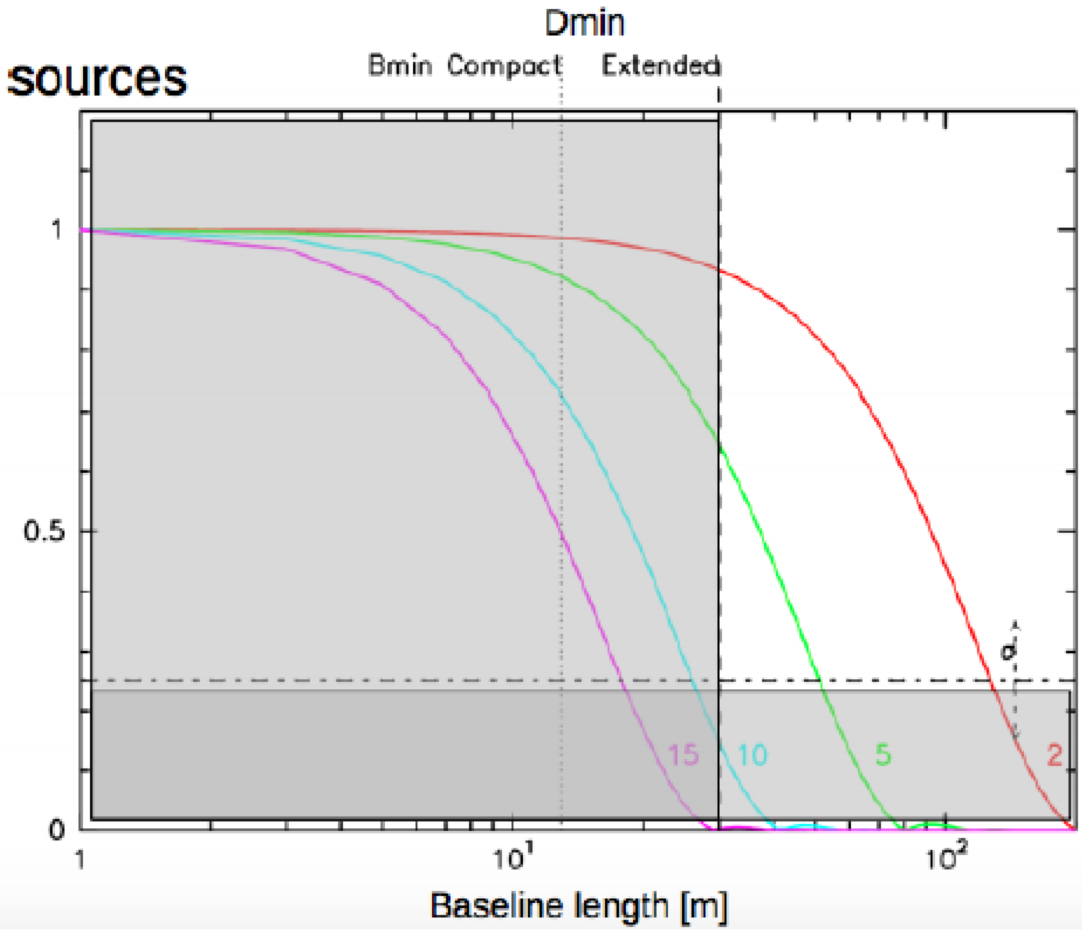

sampling uv vs. MRS the zero spacing problem

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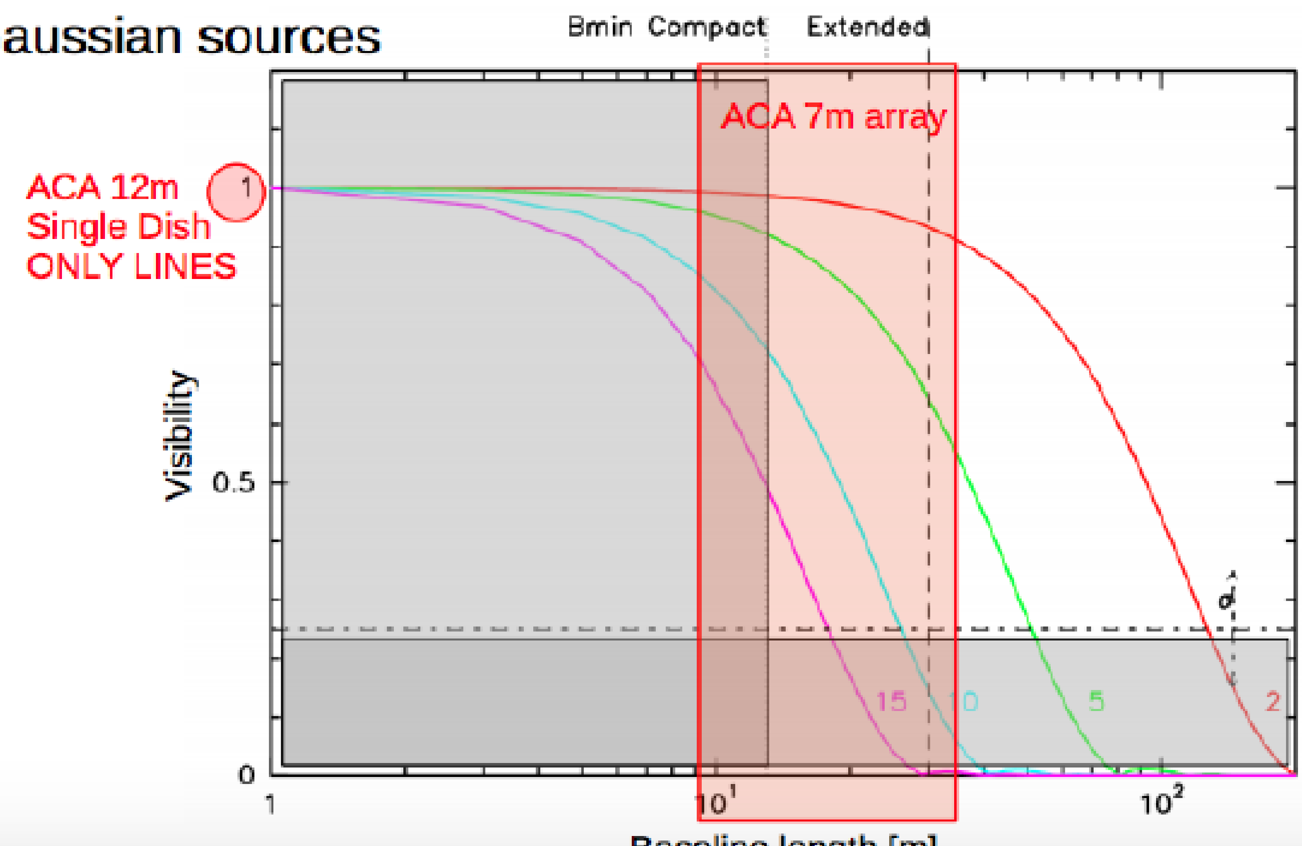


Synthesis array is `blind' to structures on angular scales both smaller and larger than the range of fringe spacings given by the antenna distribution.

FOV

$$FOV \sim \lambda/D$$

Resolution

$$\Theta_{res} \sim \lambda/B_{max}$$

Maximum scale observable

$$\Theta_{max} \sim \lambda/b_{min}$$

Sensitivity

$$\sigma \propto \frac{T_{sys}}{A_{eff}\sqrt{N(N-1)\Delta\nu\tau}}$$

T_{sys}	**System temperature**
A_{eff}	**Effective area**
N	**Number of Antennas**
$\Delta\nu$	**Bandwidth**
τ	**Observing time**

Glossary: Flux vs. Brightness

- Temperature and Fluxes (Rayleigh-Jeans)
  - S = Flux density (Jy, Jy per beam)
  - T = brightness temperature (K)
  - k Boltzmann constant
  - Ω_S solid angle (steradian)
  - θ_b HPBW of a gaussian

$$I_\nu(\theta,\varphi) = \frac{2k\nu^2}{c^2} T_B(\theta,\varphi).$$

$$S_\nu = \frac{2k\nu^2}{c^2} \int T_B d\Omega.$$

$$\left(\frac{T}{1\ \mathrm{K}}\right) = \left(\frac{S_\nu}{1\ \mathrm{Jy}}\right) \left[13.6 \left(\frac{300\ \mathrm{GHz}}{\nu}\right)^2 \left(\frac{1''}{\theta_{max}}\right) \left(\frac{1''}{\theta_{min}}\right)\right].$$

1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

Glossary: Flux vs. Brightness

- Sensitivity: depends on … a lot of things

The rms noise in the signal (sensitivity):

Tsys is the brightness temperature equivalent to the flux received from the antenna **source**, atmosphere, instrumental noise….

$$\sigma_S = \frac{2\,k\,T_{\rm sys}}{\eta_q \eta_c A_{\rm eff} \sqrt{N(N-1)\,n_p\,\Delta\nu\,t_{\rm int}}}$$

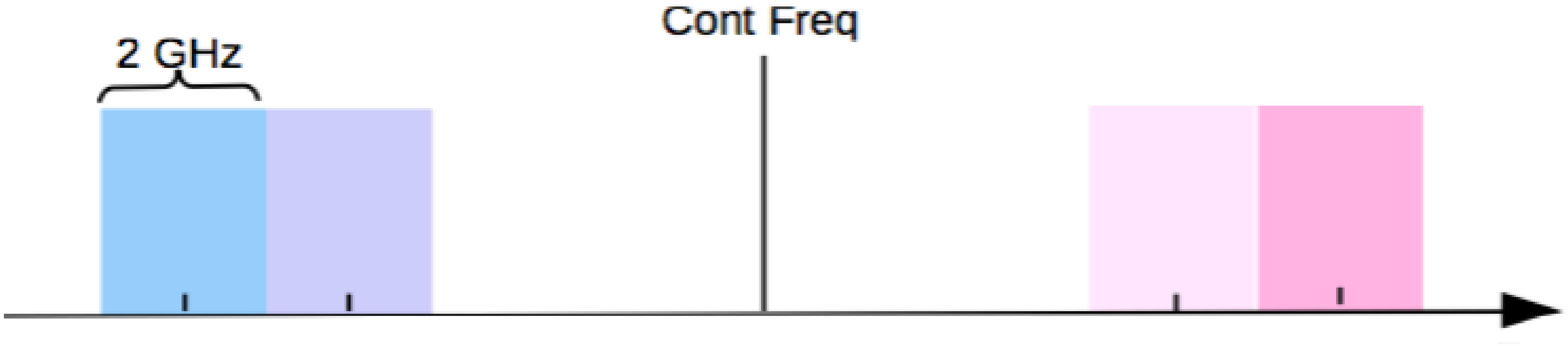
Sensitivity can be improved by:

- getting **lower Tsys** (sites with low water vapour levels)
- increasing the **collecting area**
- increasing the **bandwidth** and/or the **integration time**

The interferometric data output

(ALMA) data format—> the cube

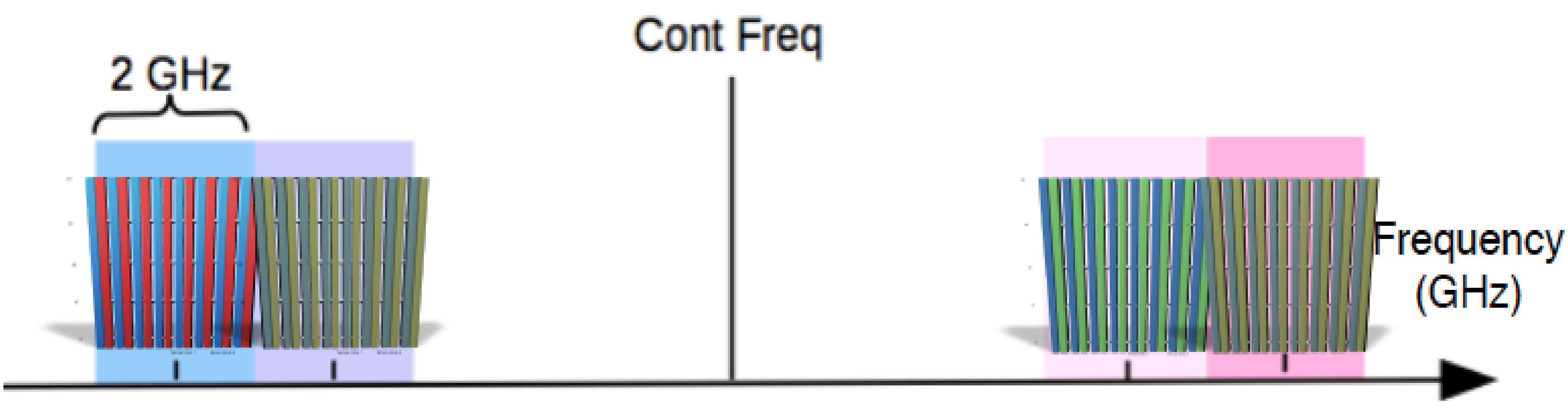
@mm wavelengths molecular spectroscopy
wide spectral range (~8GHz)
each spw divided into several channels



The interferometric data output

(ALMA) data format—> the cube

@mm wavelengths molecular spectroscopy
wide spectral range (~8GHz)
each spw divided into several channels



The interferometric data output

(ALMA) data format—> the cube

From each channel, one uv-plane/image is produced

Spectral line observations have up to 3840 channels. The highest spectral resolution achieavable is 30 kHz.