Short intro to Interferometry

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Ideas and slides borrowed from

Radio-Astronomy course - UNI Bologna (Rosita Paladino)
http://www.alma.inaf.it/index.php/Courses

IRAM interferometry school
http://www.iram-institute.org/EN/content-page-331-7-67-331-0-0.html

NRAO interferometry school
https://science.nrao.edu/science/meetings/2016/15th-synthesis-imaging-workshop

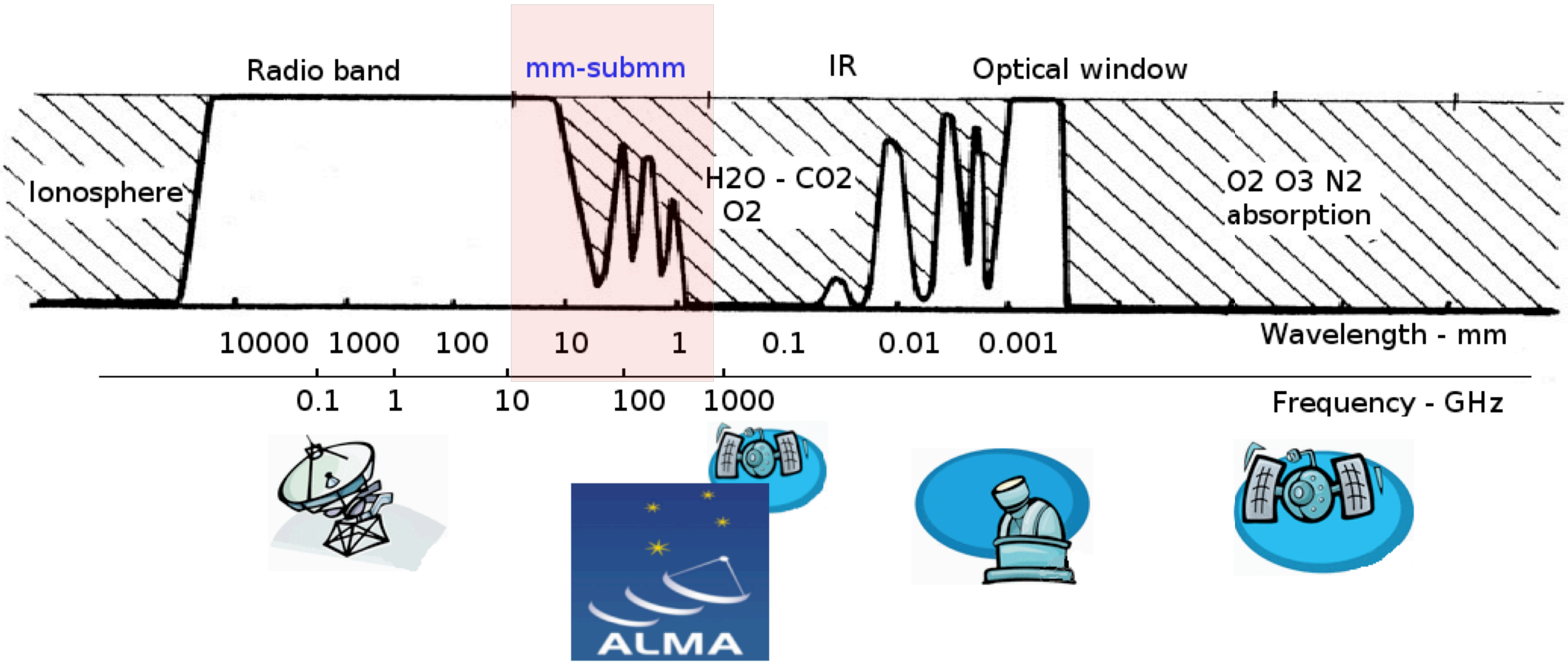
LOFAR school
http://www.astron.nl/lofarschool2016/

European Radio interferometry (ERIS) school
https://www.eso.org/sci/meetings/2015/eris2015.html

Synthesis Imaging in Radio Astronomy: II – The "White Book"

Virtual Radio Interferometer
http://www.narrabri.atnf.csiro.au/astronomy/vri.html

submm interferometry....



~ 80-900 GHz
~ 3-0.3 mm

Motivation: angular resolution of observations

Angular resolution of diffraction-limited telescope is

$$\theta \sim \lambda/D \text{ radians}$$

where D is the diameter of the telescope and λ is the wavelength of observations

For example, Hubble Space Telescope

- $\lambda \sim 1\mu m$ and D of 2.4 m → $\theta \sim 0.13$ arcsec

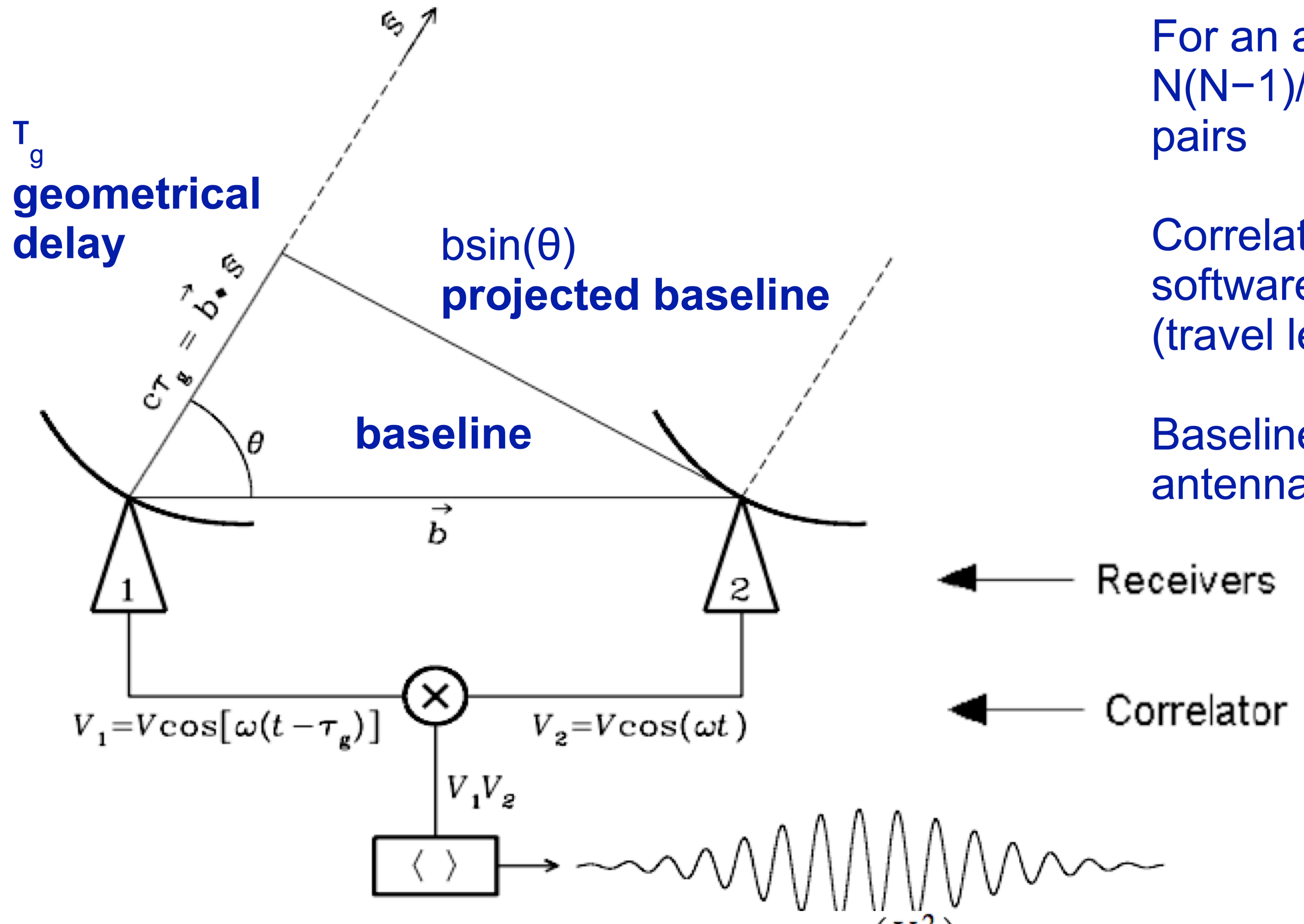
To reach that angular resolution for **λ=1 mm observations**
→ it is necessary a **2 km-diameter dish**!

Instead, we use arrays of smaller dishes to achieve the same angular resolution at radiofrequencies synthesizing a large aperture by combining signals collected by separated small apertures

This is interferometry

Interferometry: the basics

quasi-monochromatic interferometer centered at narrow frequency range ν=ω/2π



$$R = \langle V_1 V_2 \rangle = \left(\frac{V^2}{2}\right) \cos(\omega \tau_g) .$$

The basic interferometer is a pair of radio telescopes whose voltage outputs are correlated (multiplied and averaged).

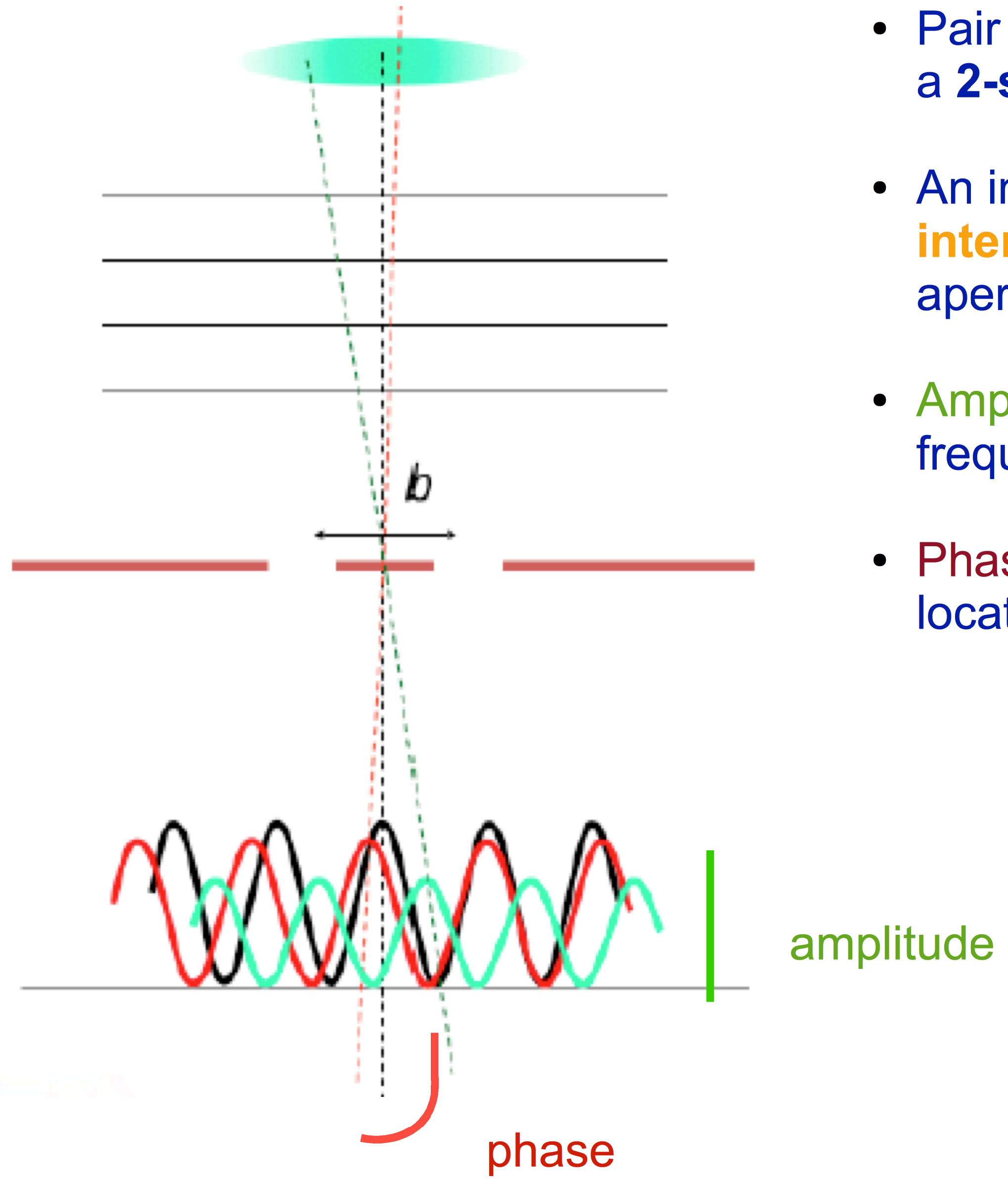
For an array of N antennas, there are N(N−1)/2 independent interferometer pairs

Correlator compensates in the software for the geometrical delay (travel length)

Baseline is the distance between the antennas

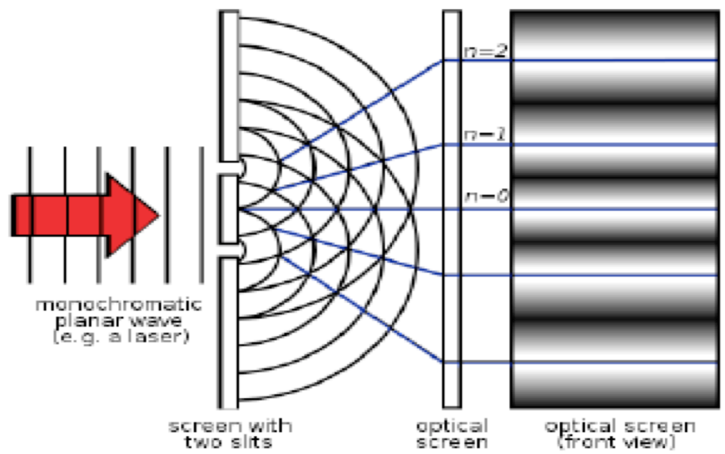
Interferometry: the basics

Extended source with bright component at the center of phases and fainter one far away



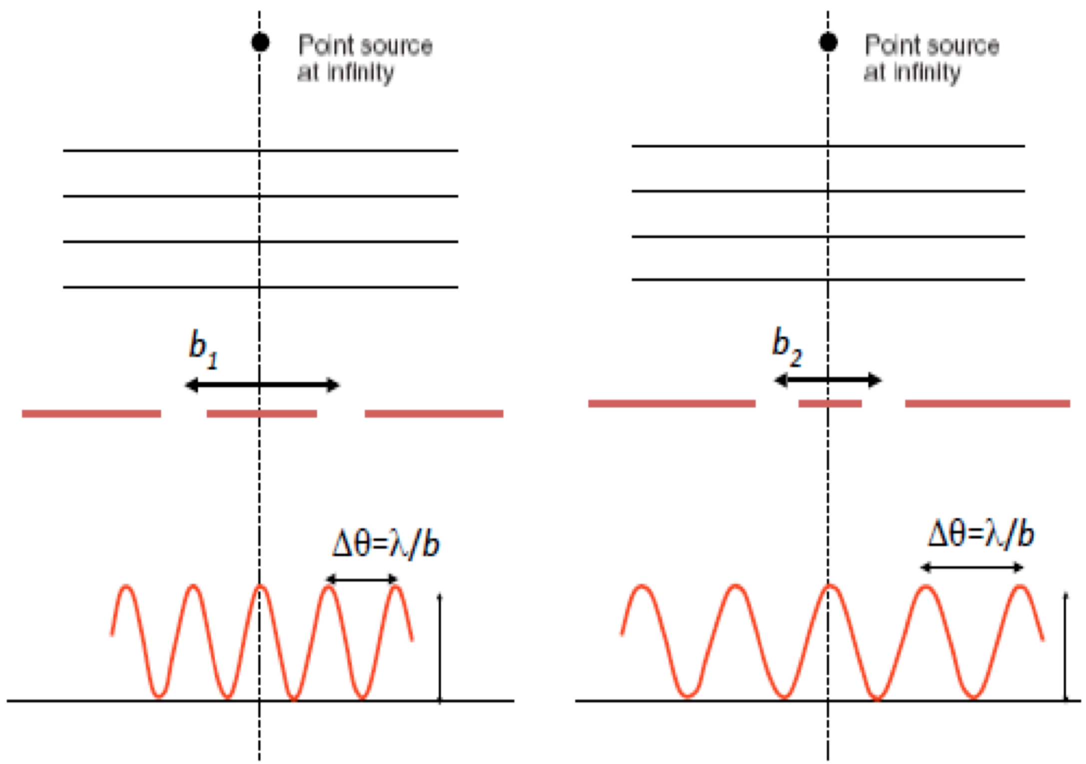
- Pair of radio telescopes work like a **2-slit Young's experiment**
- An interferometer of N antennas measures the **interference pattern** produced by pairs of apertures –> Visibility
- Amplitude tells "how much" of a certain frequency component
- Phase tells "where" the component is located

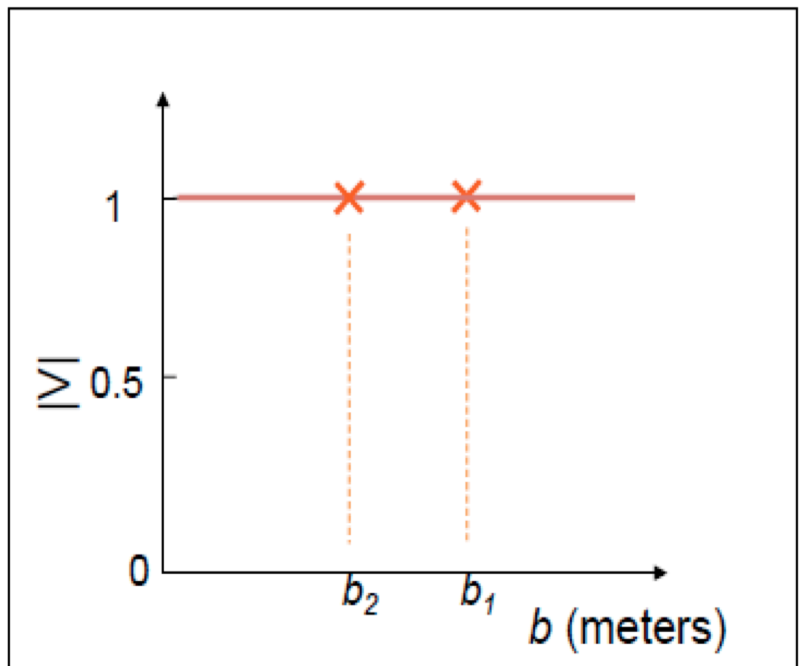
2-slit experiment



Interferometry: the basics

Point-like source at the center of phases seen by an array with b1 and b2 baselines....





$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

Amplitude constant over b and phase constant = 0

→ potential good calibrator (depending on flux level)

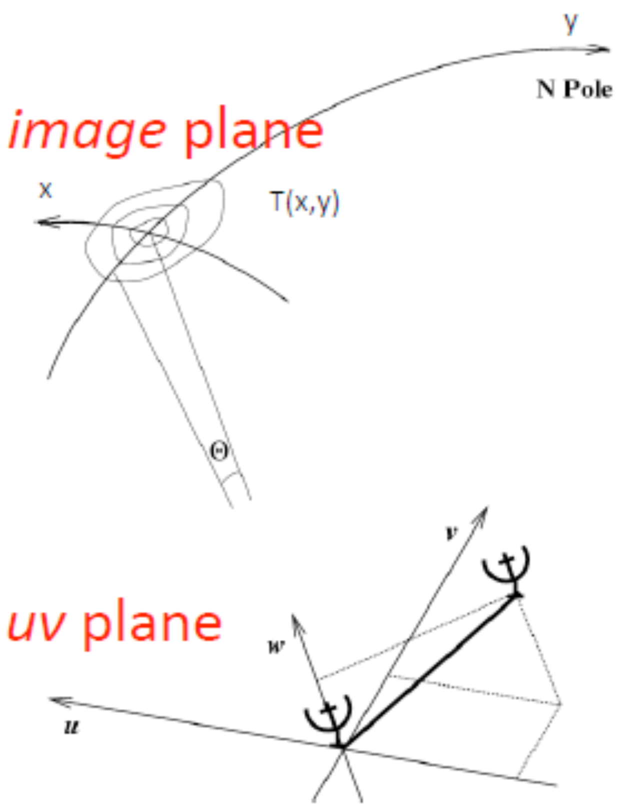
Visibility and Sky Brightness

The Fourier Theory (FT) states that any well behaved signal (including imaging) can be expressed as sum of sinusoids.

The FT relates the measured interference pattern to the source brightness:

- **$V = A e^{-i\phi}$** where A is amplitude and ϕ is the phase
- For small fields of view, the **complex visibility V(u,v) is the 2D Fourier Transform of the brightness on the sky (T(x,y))**

(van Cittert-Zernike theorem)



Fourier space/domain

$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dxdy$$

$$T(x,y) = \int\int V(u,v) e^{-2\pi i(ux+vy)} dudv$$

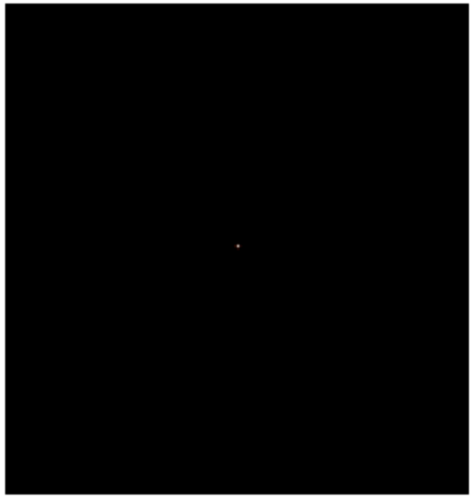
Image space/domain

- **u,v (wavelengths)** are spatial frequencies in E-W and N-S, i.e the E-W and N-S component of the projected baselines
- **x,y (rad)** are angles in tangent plane related to a reference position in the E-W and N-S directions

Some 2D Fourier Transform Pairs

T(x,y) — Amp{V(u,v)}

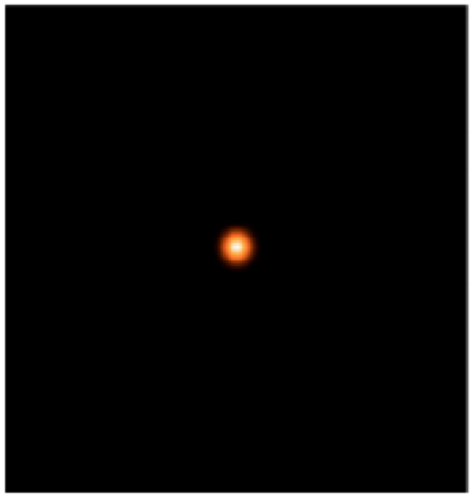
δ Function



⇌

Constant

Gaussian

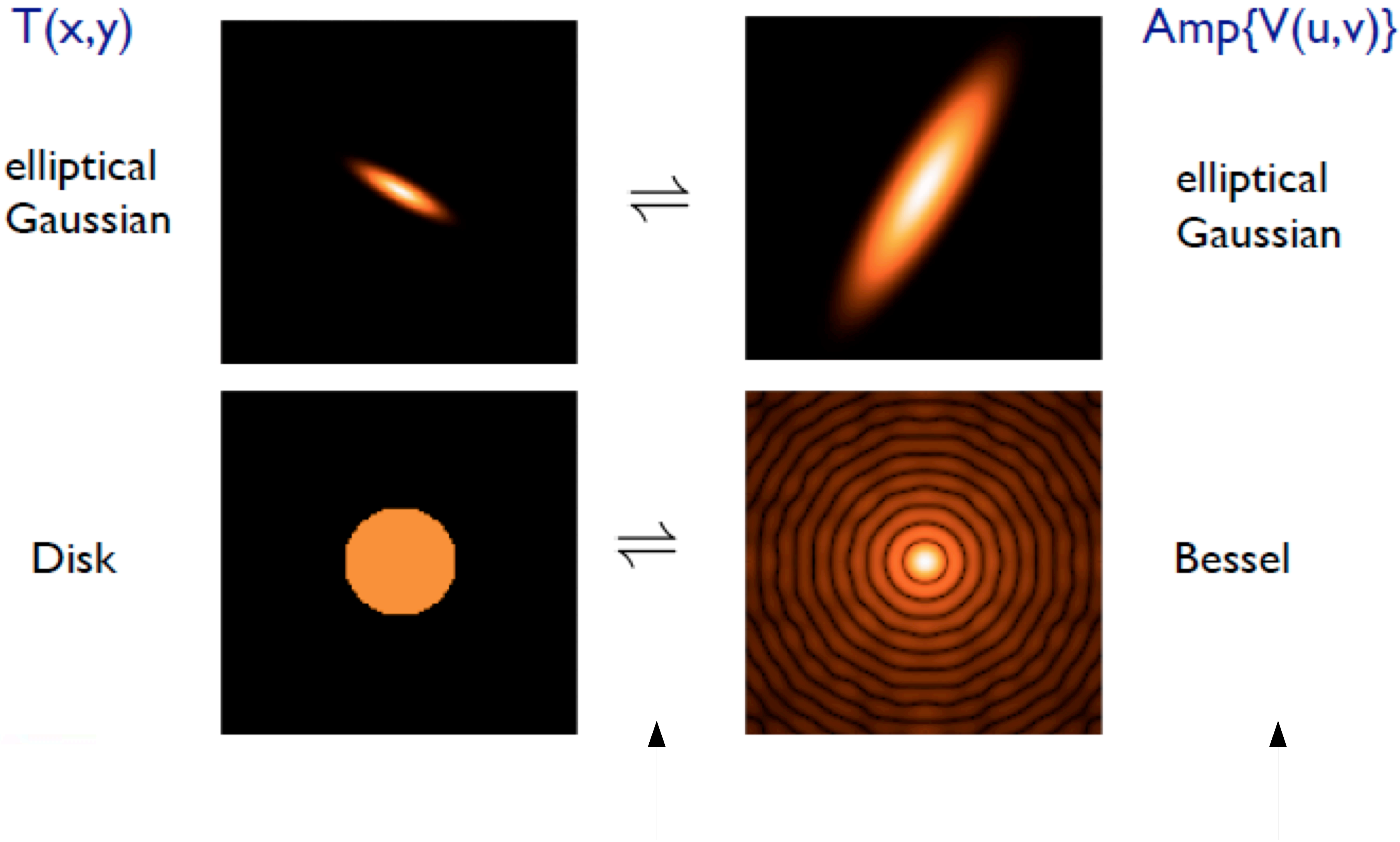


⇌

Gaussian

narrow features transform to wide features (and vice-versa)

2D Fourier Transform Pairs



Imaging from V(u,v) to T(x,y)

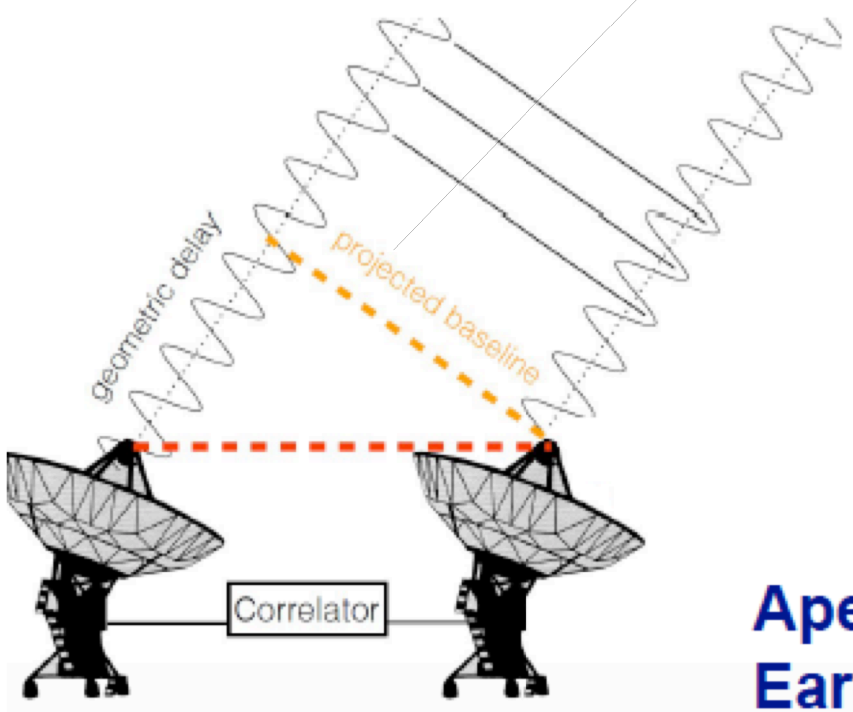
Calibration on V(u,v)

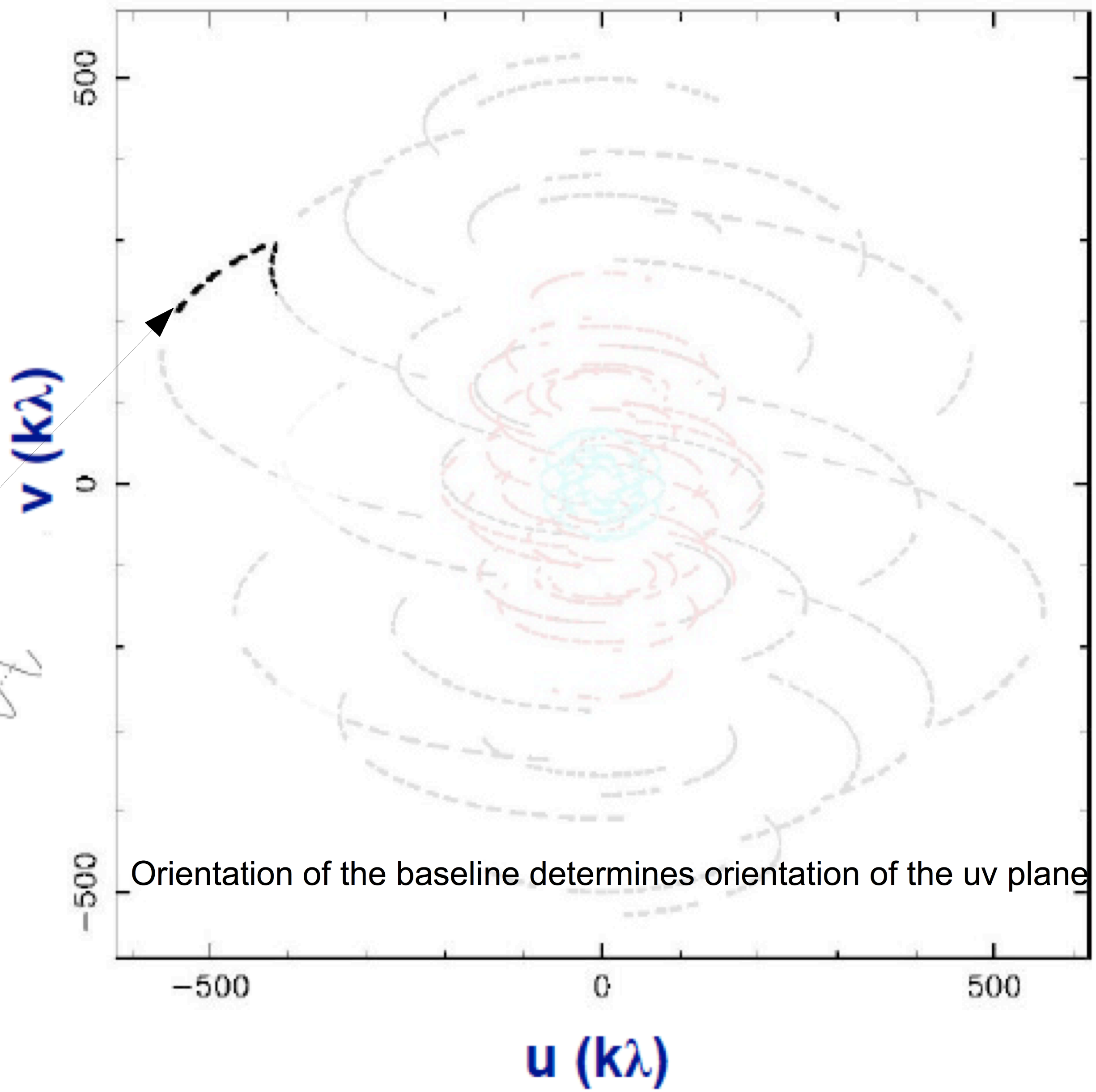
V(u,v) and baseline

V=V(u,v) is
complex and Hermitian
→ V(-u, -v)=V*(u,v)

1 V for each integration time,
for each channel
for each baseline

Each antenna pair → a point in the (u,v) plane

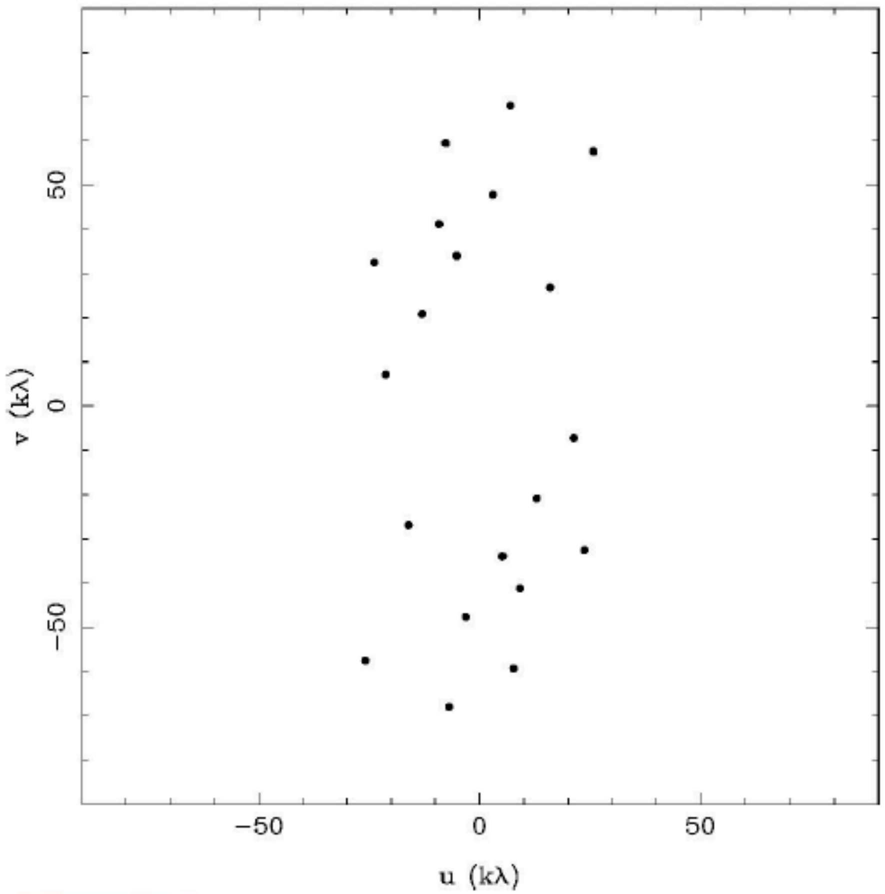


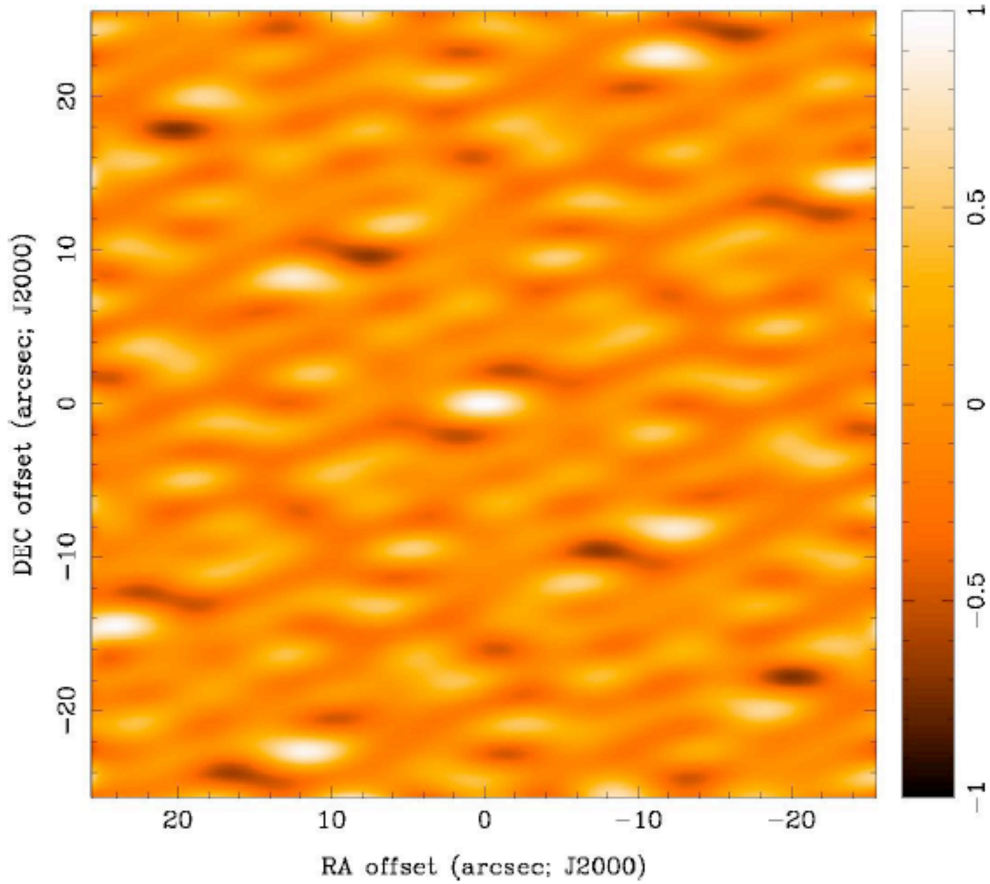


Aperture synthesis:
Earth rotation helps covering the uv plane

PSF shape vs N ants, time

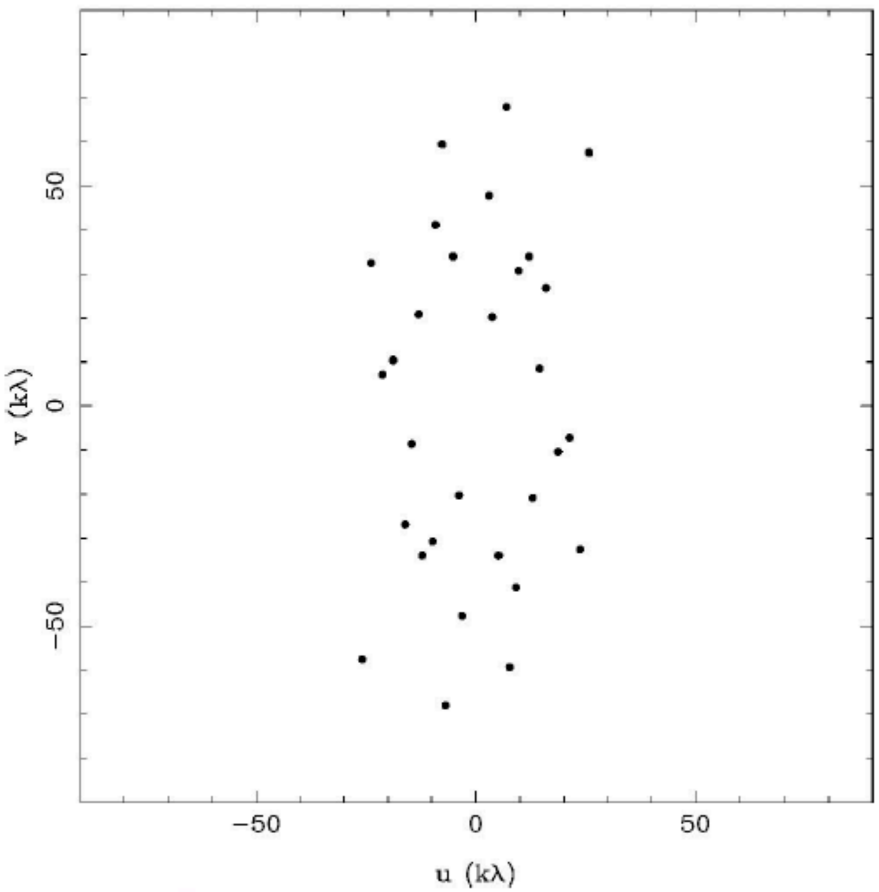
5 Antennas, 1 Sample

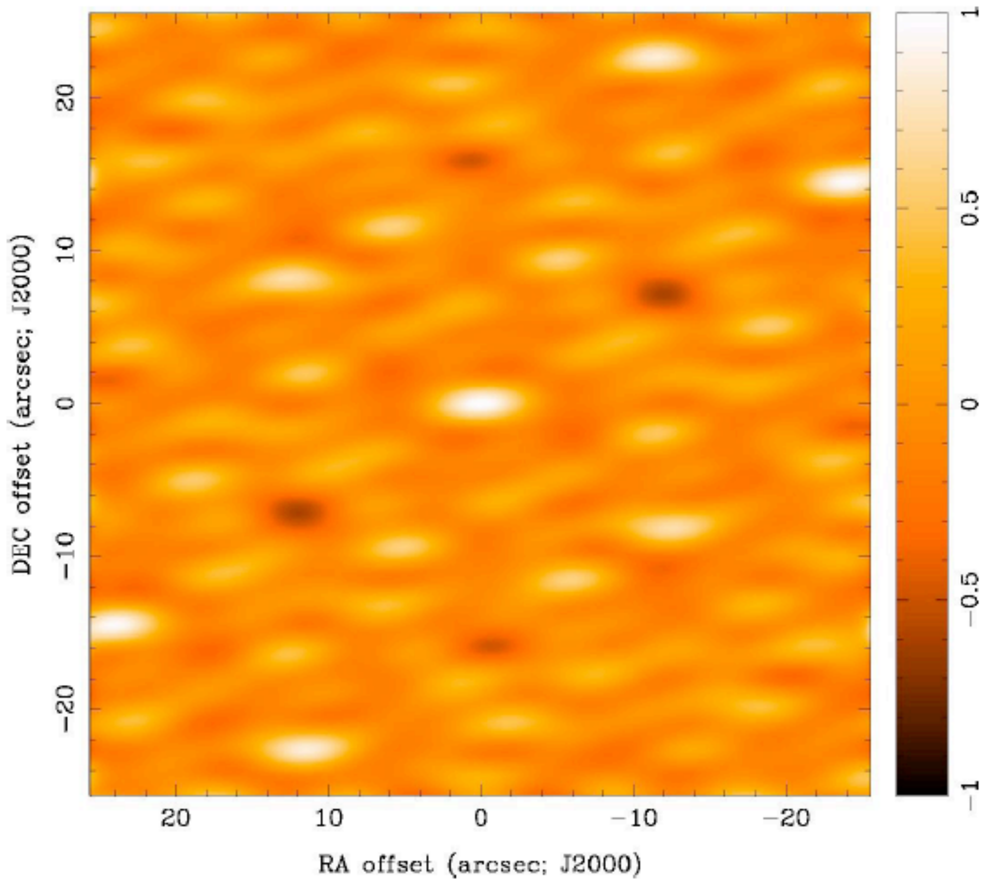




PSF shape vs N ants, time

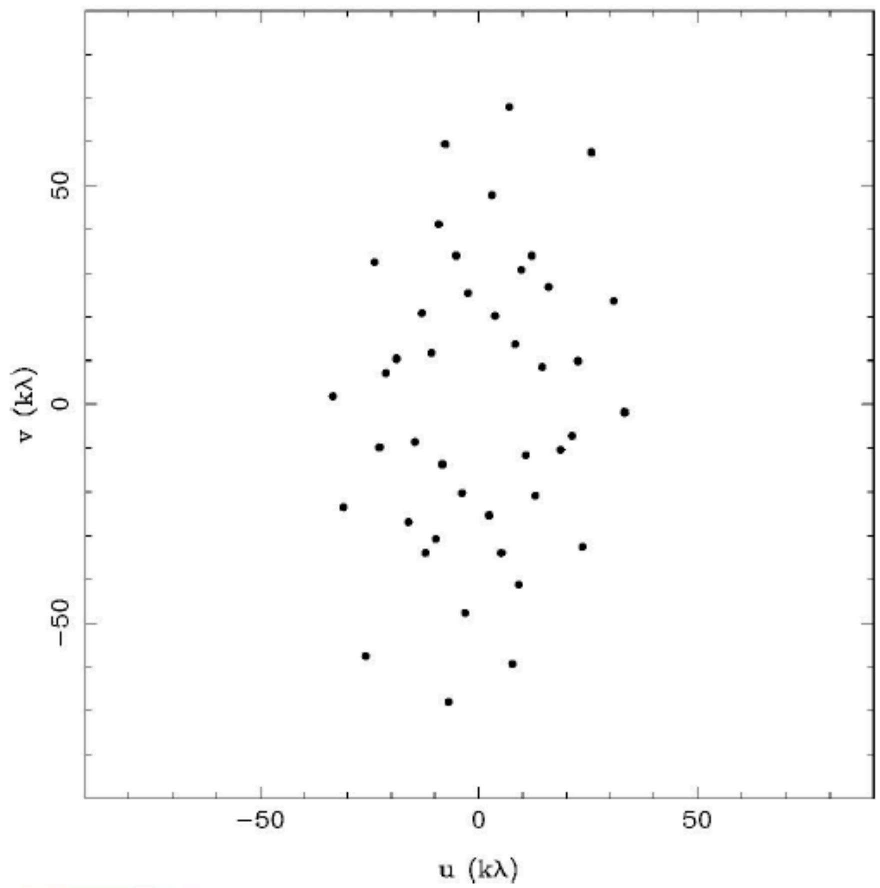
6 Antennas, 1 Sample

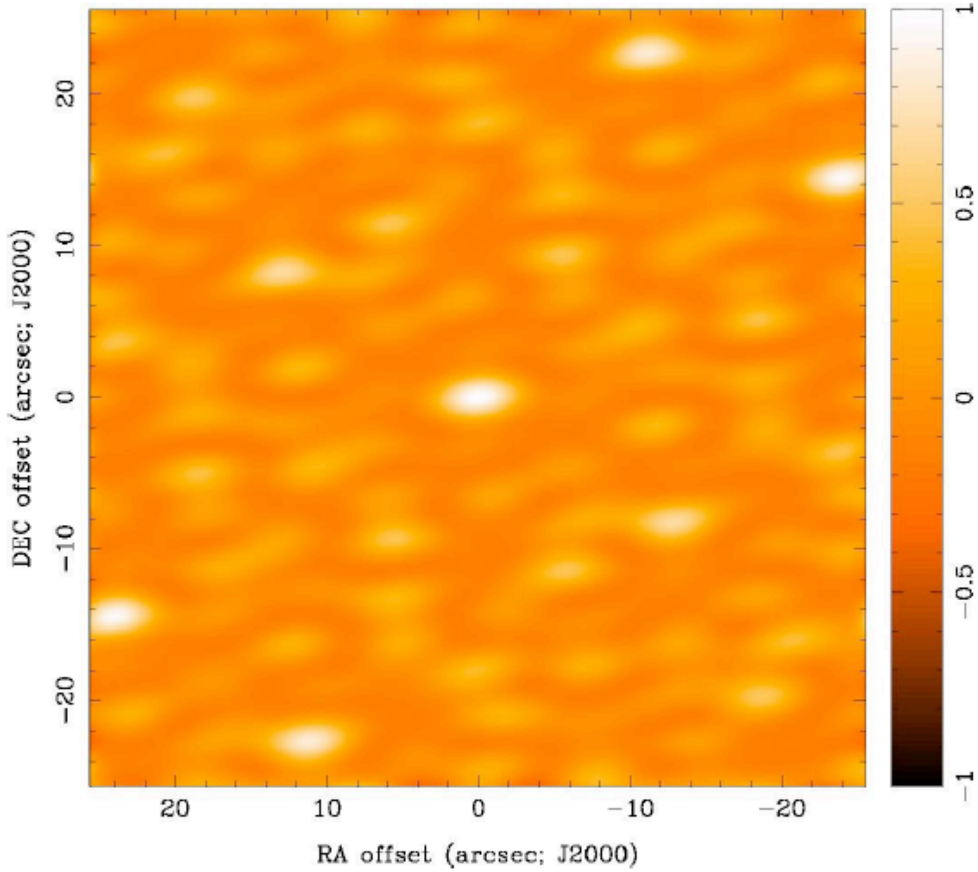




PSF shape vs N ants, time

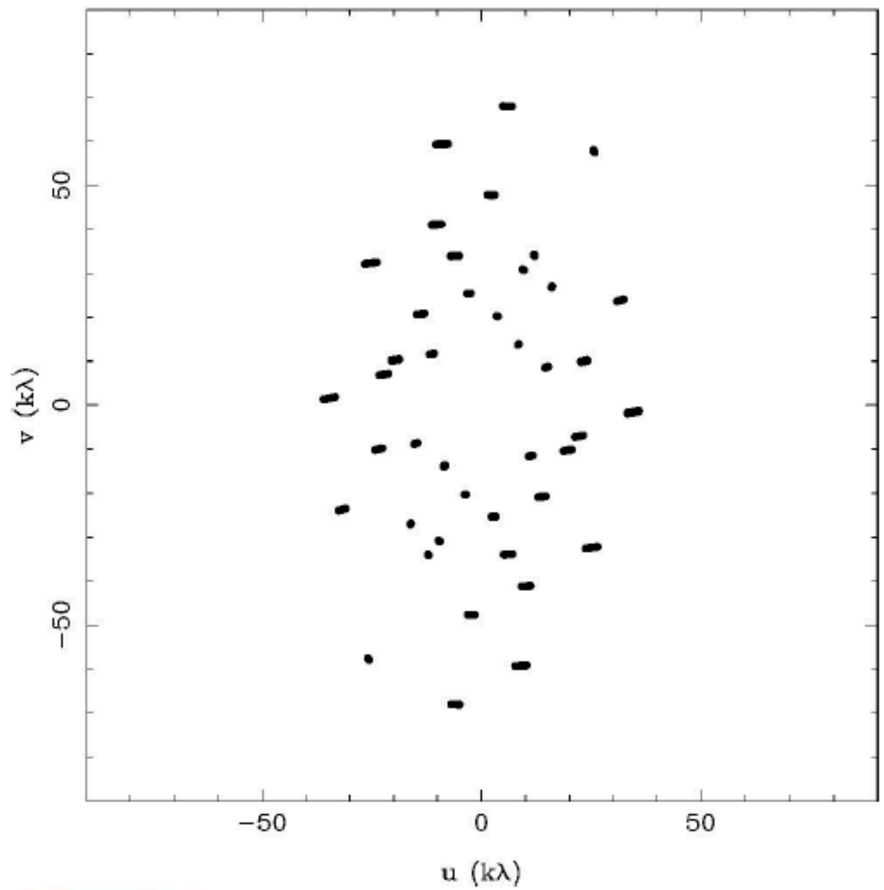
7 Antennas, 1 Sample

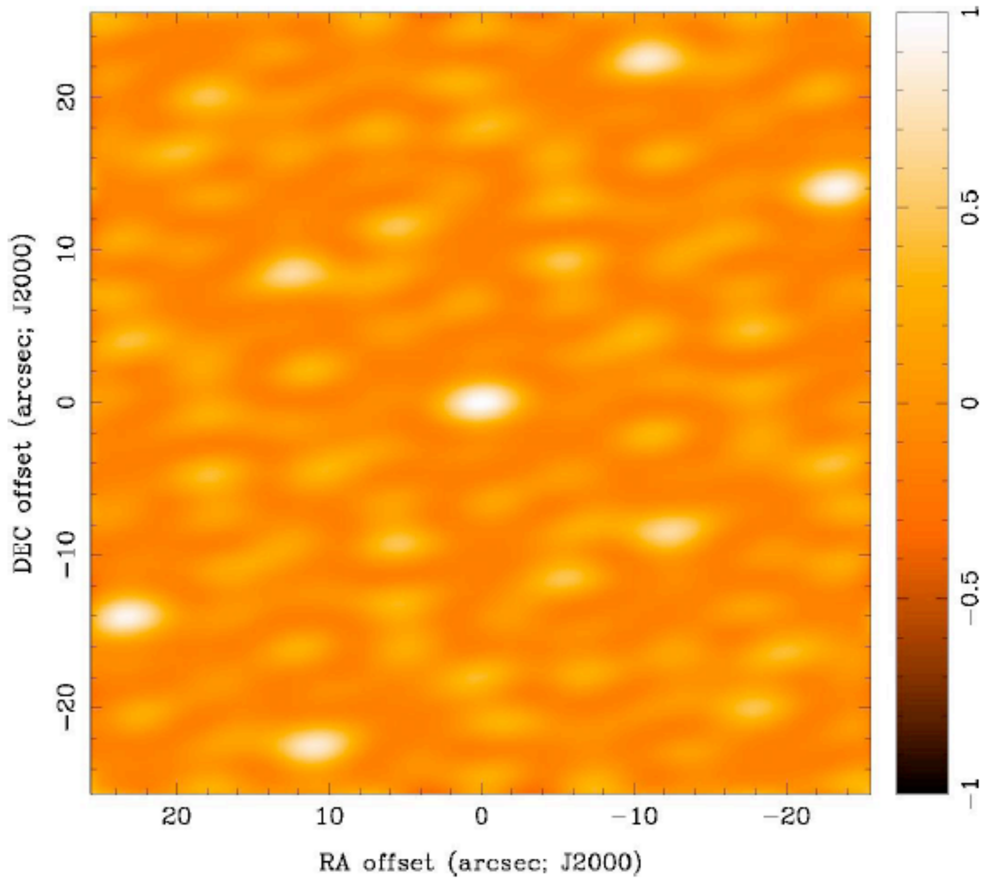




PSF shape vs N ants, time

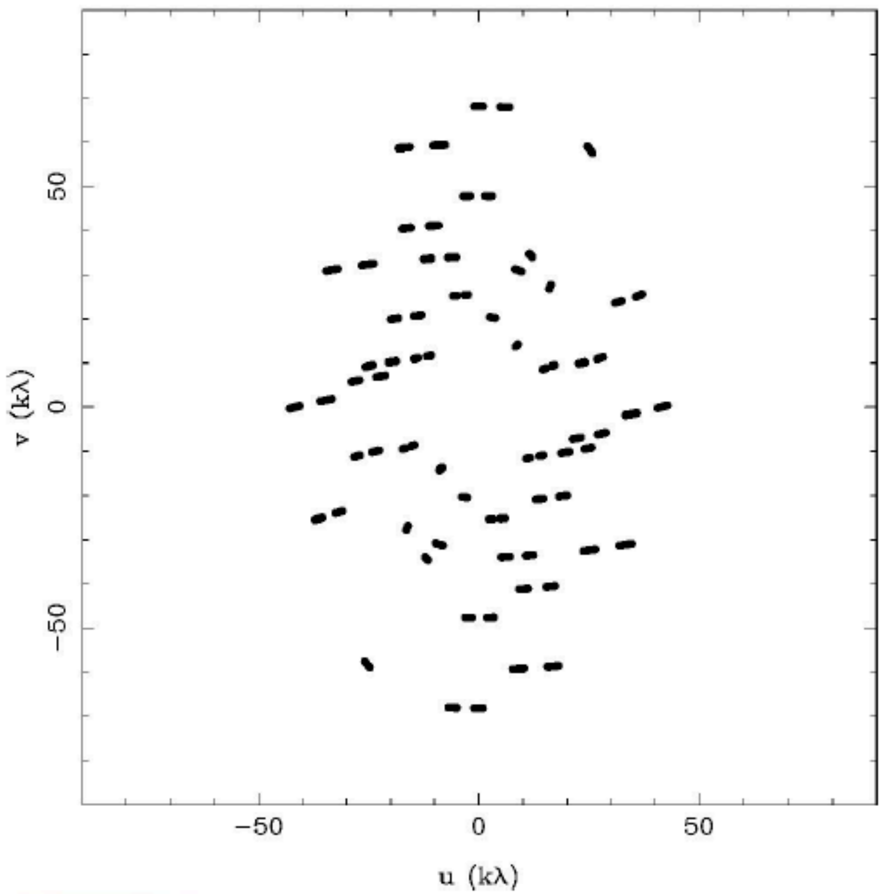
7 Antennas, 10 min

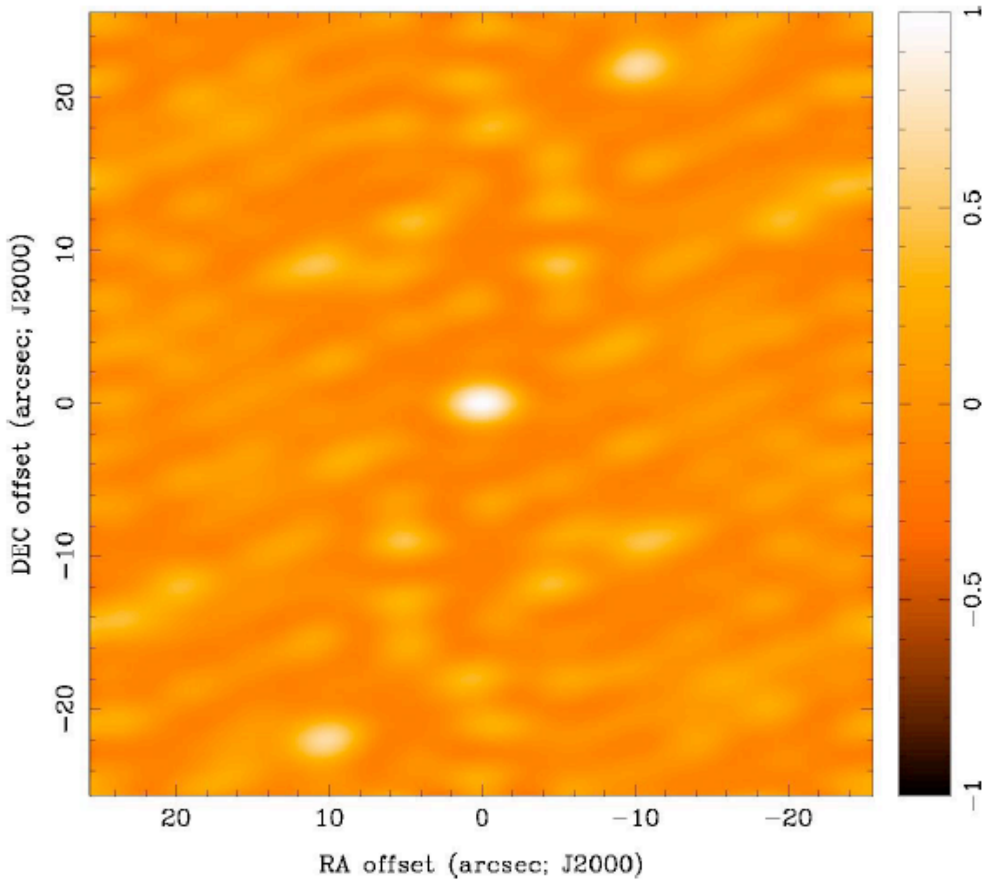




PSF shape vs N ants, time

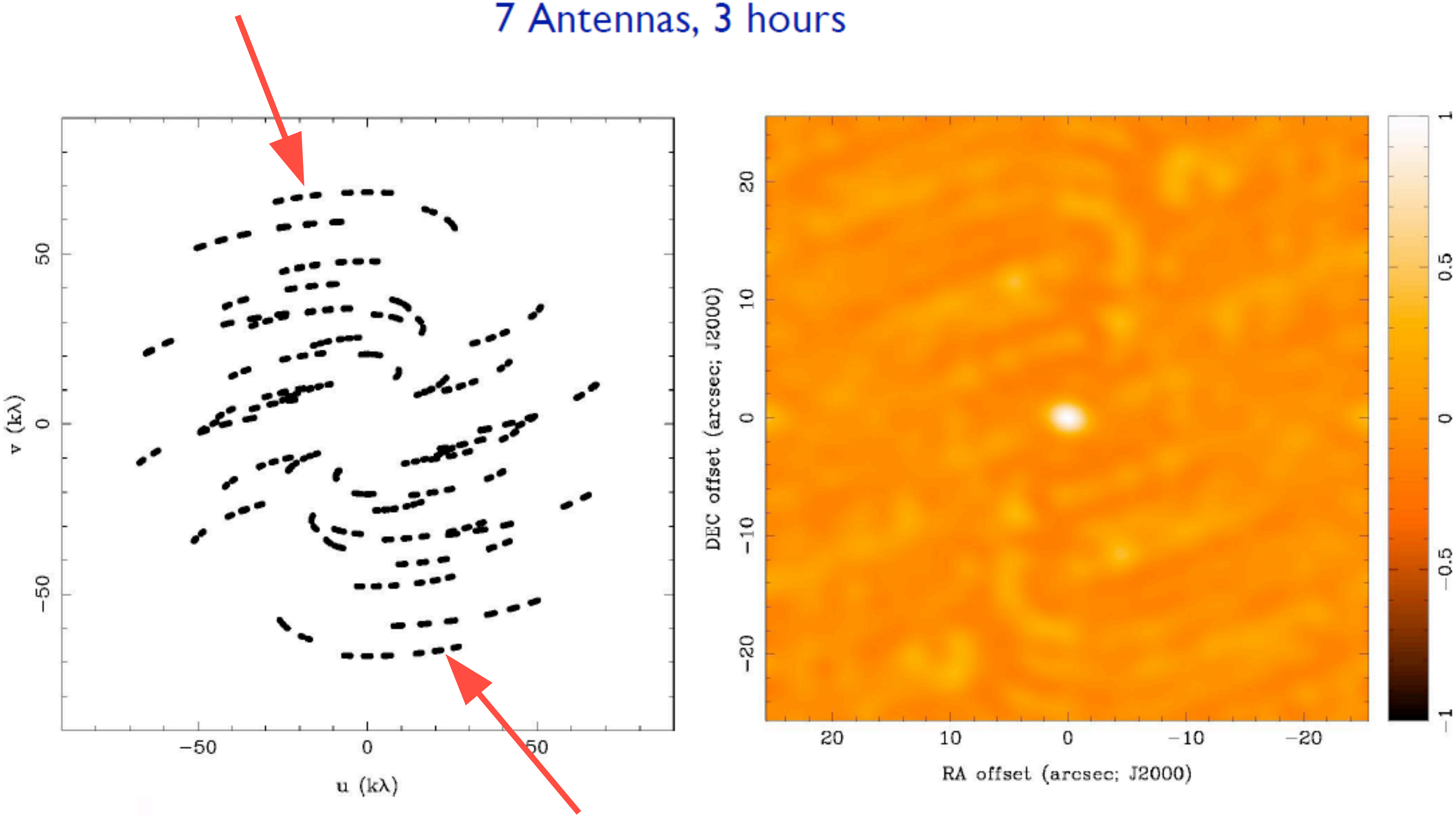
7 Antennas, 2 x 10 min





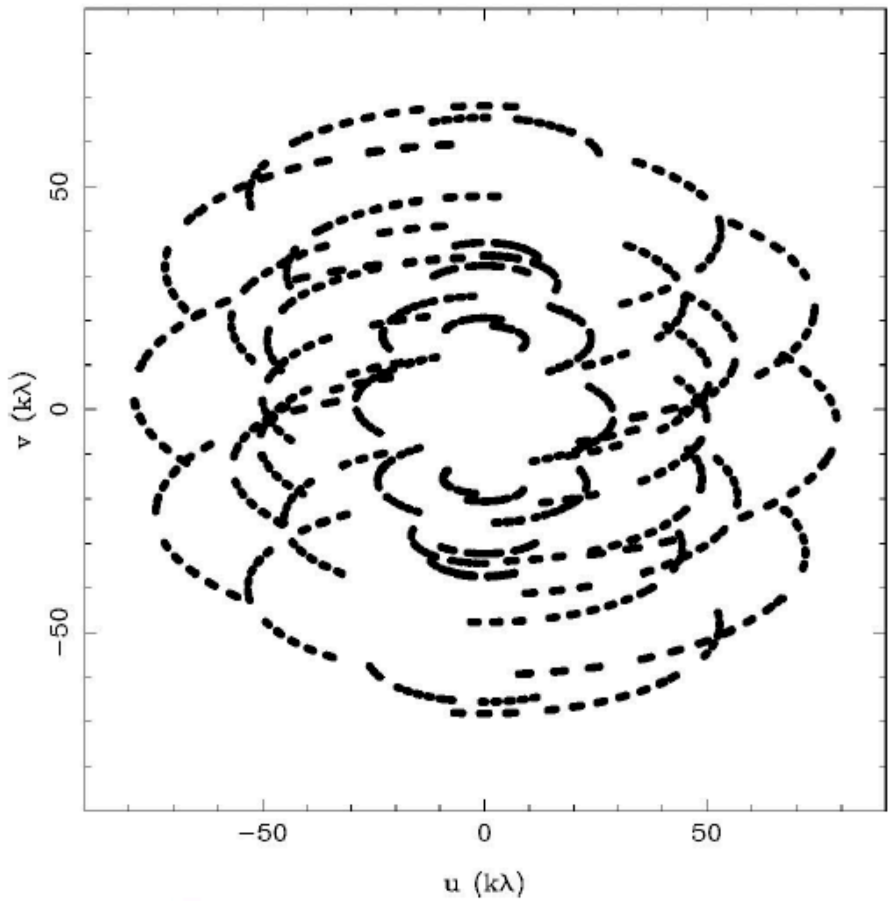
PSF shape vs N ants, time

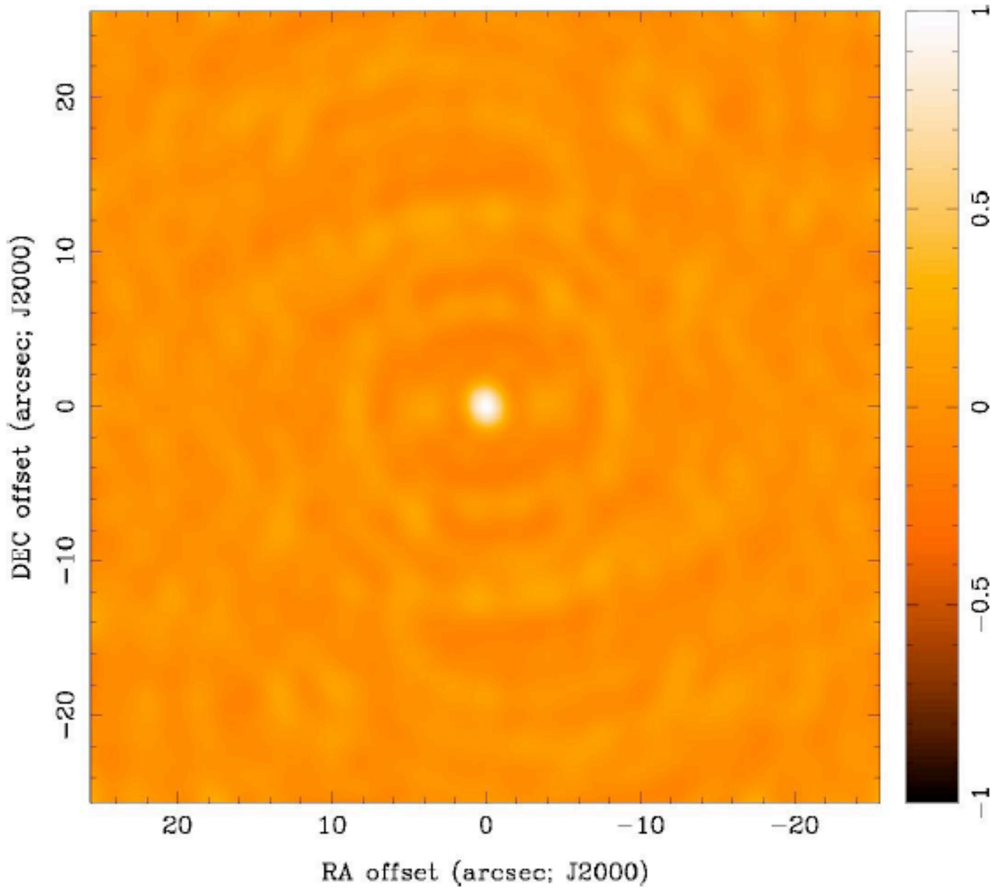
7 Antennas, 3 hours



PSF shape vs N ants, time

7 Antennas, 8 hours





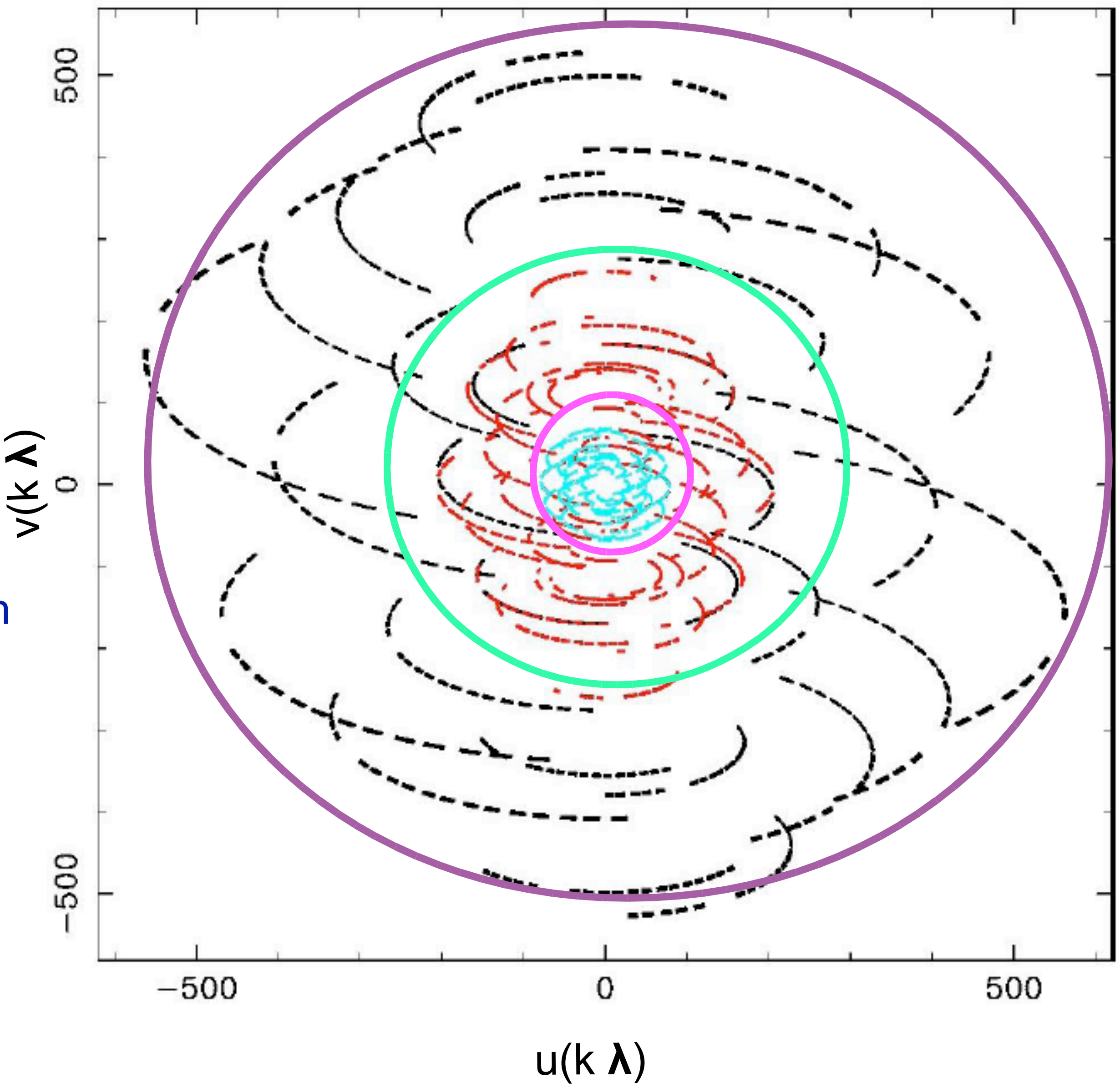
ALMA has a quasi instantaneous (u,v) coverage

V(u,v) and baseline

$\Theta_{res} \sim \lambda / B_{max}$
where B_{max} is the maximun baseline
(uv distance)

Long (u,v) distance:
long baseline,
measure more compact emission

Small (u,v) distance:
short baseline,
measure more extended emission



V(u,v) and baseline

$\Theta_{res} \sim \lambda / B_{max}$

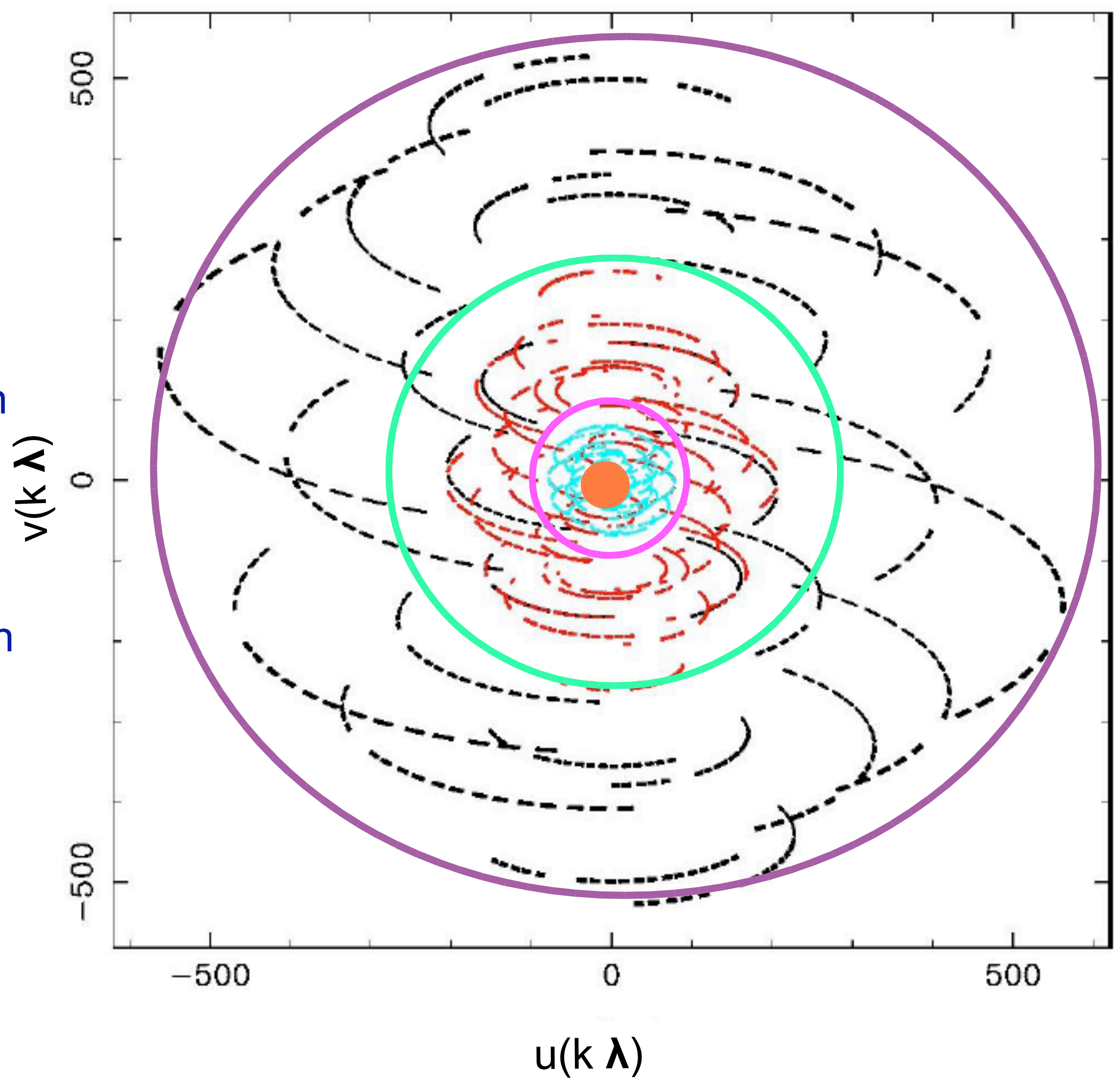
where B_{max} is the maximun baseline (uv distance)

Long (u,v) distance:
- long baseline,
- measure more compact emission

Small (u,v) distance:
- short baseline,
- measure more extended emission

Zero spacing:
- missing
- limit on observable largest scale

$\Theta_{MRS} \sim \lambda / B_{min}$



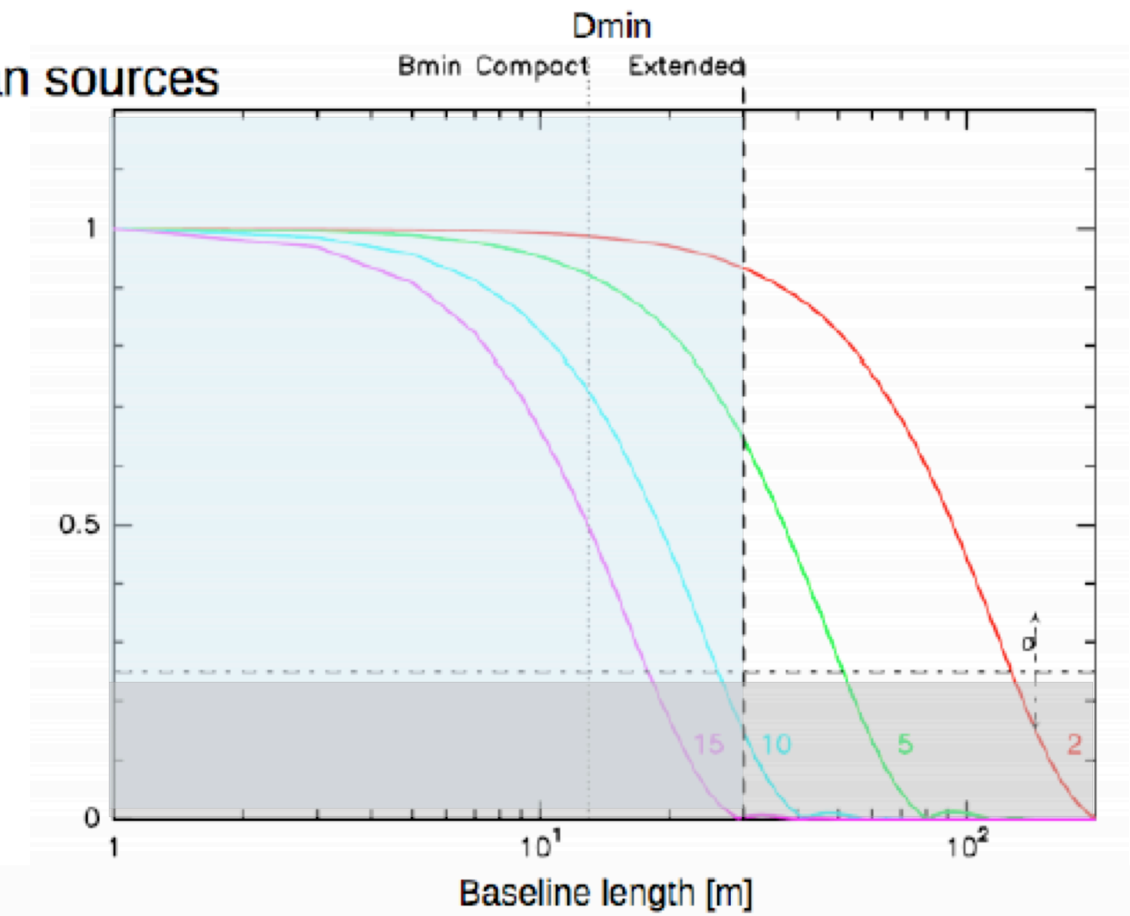
(u,v) sampling vs Maximum recoverable scale (MRS)

Zero spacing missing in interferometry

- Filtering of large scale emission

$$\Theta_{MRS} \sim \lambda / B_{min}$$

- Gaussian sources



(u,v) sampling vs Maximum recoverable scale (MRS)

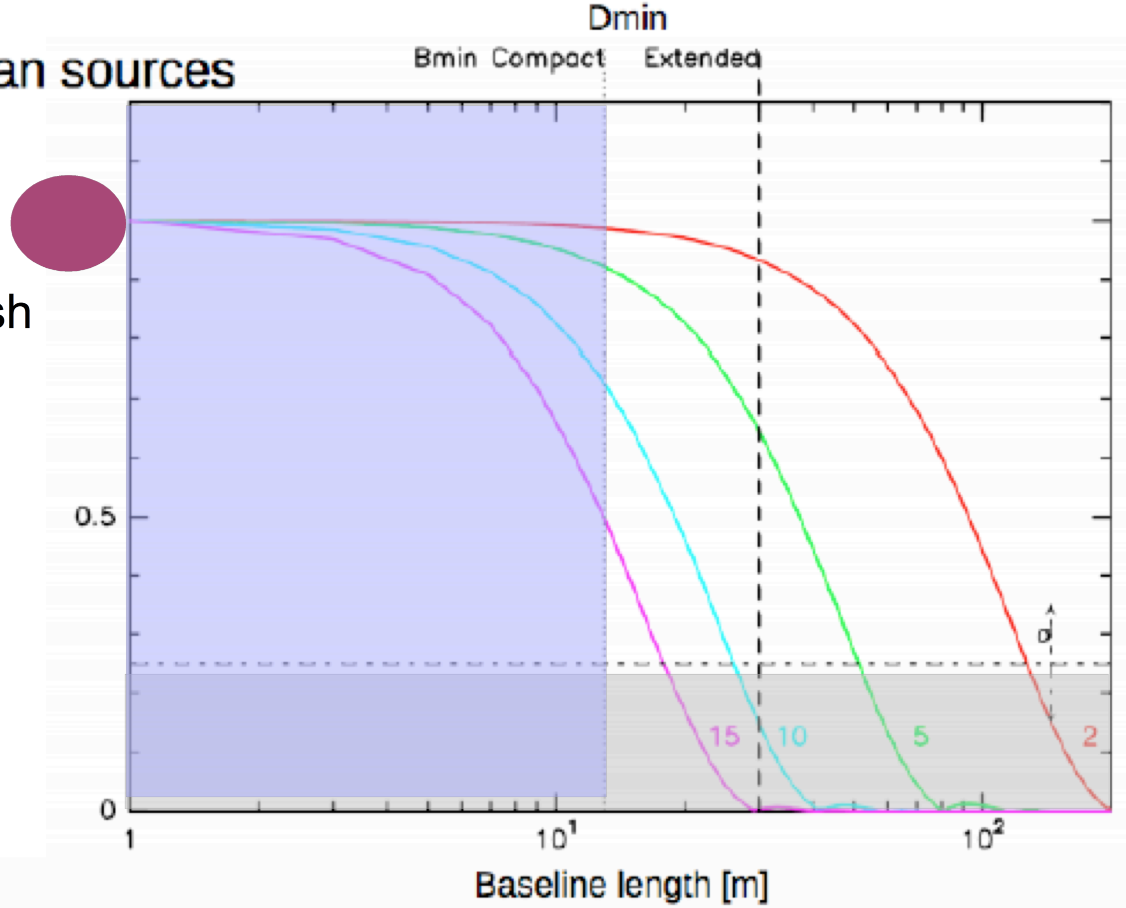
Zero spacing missing in interferometry

- Filtering of large scale emission

$$\Theta_{MRS} \sim \lambda / B_{min}$$

→ We need total power observations to recover the zero spacing flux

- Gaussian sources



Interferometry sensitivity

For an interferometer :

$$\sigma_S \approx \frac{2\,k\,T_{sys}}{A_{eff}\,\sqrt{n(n-1)\times\Delta\nu\times\eta_{pol}\times t_{int}}}\ [\mathrm{Jy}] \qquad \sigma_T = \frac{\sigma_S\,\lambda^2}{2\,k\,d\Omega_{array}}\ [\mathrm{K}]$$

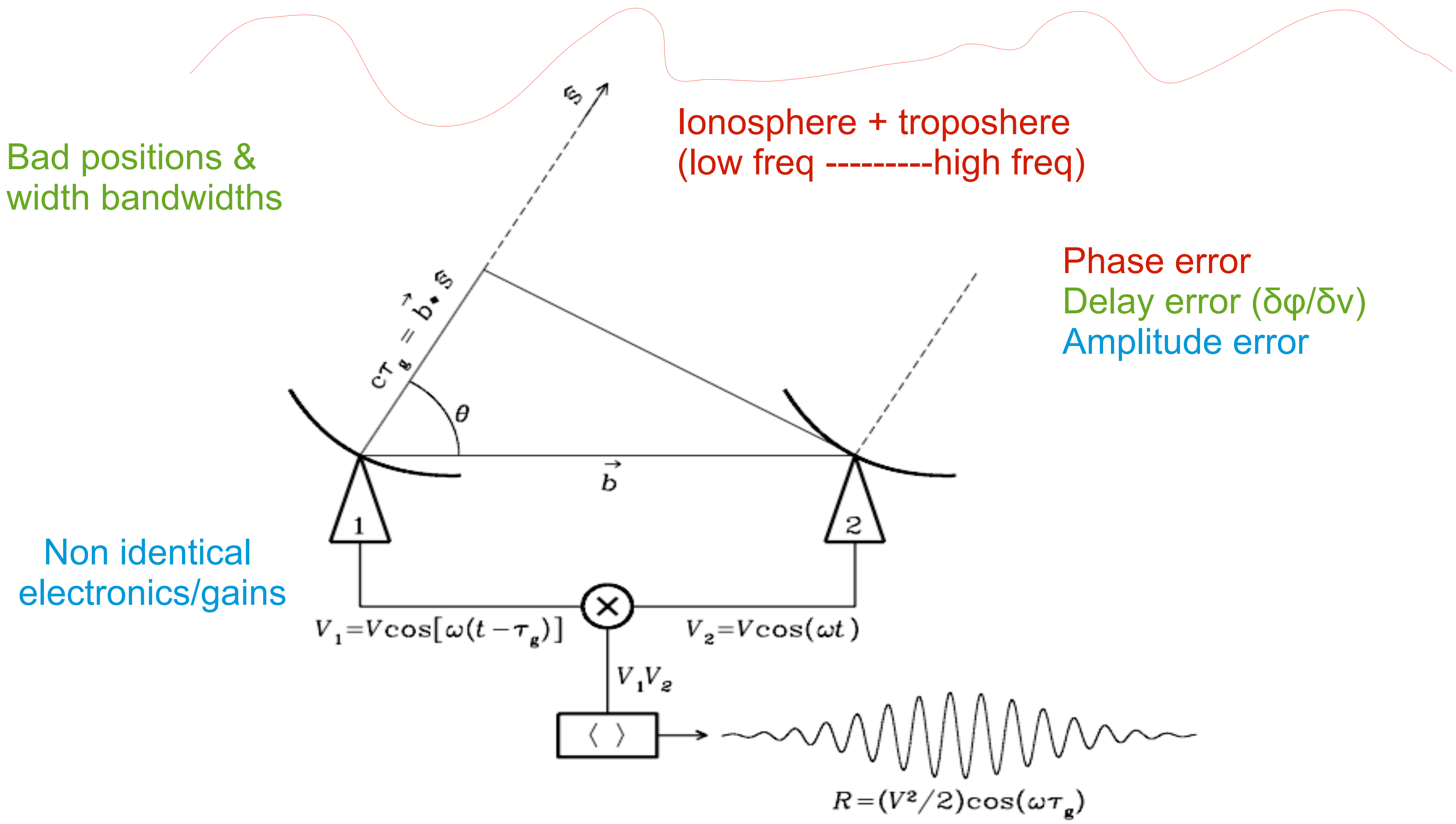
where :

A_{eff}	is the effective collecting area
$n(n-1)$	is the number of baseline and n the number of antenna
$\Delta\nu$	is the bandwidth
η_{pol}	= 1 for single polarisation and 2 for dual polarisation
t_{int}	is the integration time
$d\Omega_{array}$	is the synthesized beam, i.e. $d\Omega_{array} \approx 1.14\,\frac{\lambda^2}{B^2}$
T_{sys}	is the system temperature ($=T_{atm} + T_{rx}$)

Sensitivity can be improved:

- getting lower T_{sys} (= lowering instrumental noise or atmospheric noise)
- increasing the collecting area and/or # of antennas
- increasing the bandwidth and the observing time

Calibration in interferometry: why ?



Calibration in interferometry: what and how?

$$V_{ij}^{obs} = G_{ij} V_{ij}^{true}$$

$$V_{ij}^{obs}(A,\varphi) = G_i G_j V_{ij}^{true}(A,\varphi)$$

$$G_i = K_i B_i J_i D_i E_i P_i T_i F_i$$

where K = geometric compensation, B = Bandpass response, J= electronic gains, D=polarization leakage, E= antenna voltage pattern, P= parallactic angle,T= troposphere effects,F=ionospheric faraday rotation

$$G_i(v,t) = B_i(v) J_i(t)$$

- Calibration of amplitude and phase $V_{ij} = V_{ij}(A,\varphi)$ vs time and frequency
- Antenna-based effect $G_{ij} = G_i G_j$
- Different calibration terms are indipendent $G_i = K_i B_i J_i D_i E_i P_i T_i F_i$
- Temporal dependence and frequency dependence are only lightly coupled so their variations can be determined independently or at least iteratively $G_i(v,t) = B_i(v) J_i(t)$
- Observations of sources which visibilites $V^{true,\ known}$ are known (calibrators)

$$G_{ij} = V_{ij}^{true,known} / V_{ij}^{obs}$$

→ Calibrator is usually point source at the phase center (amplitude constant and zero phase)
→ closure phases and amplitudes

Calibration in interferometry: observational strategy in the mm

We need to calibrate A, Φ vs t, ν of Visibilities

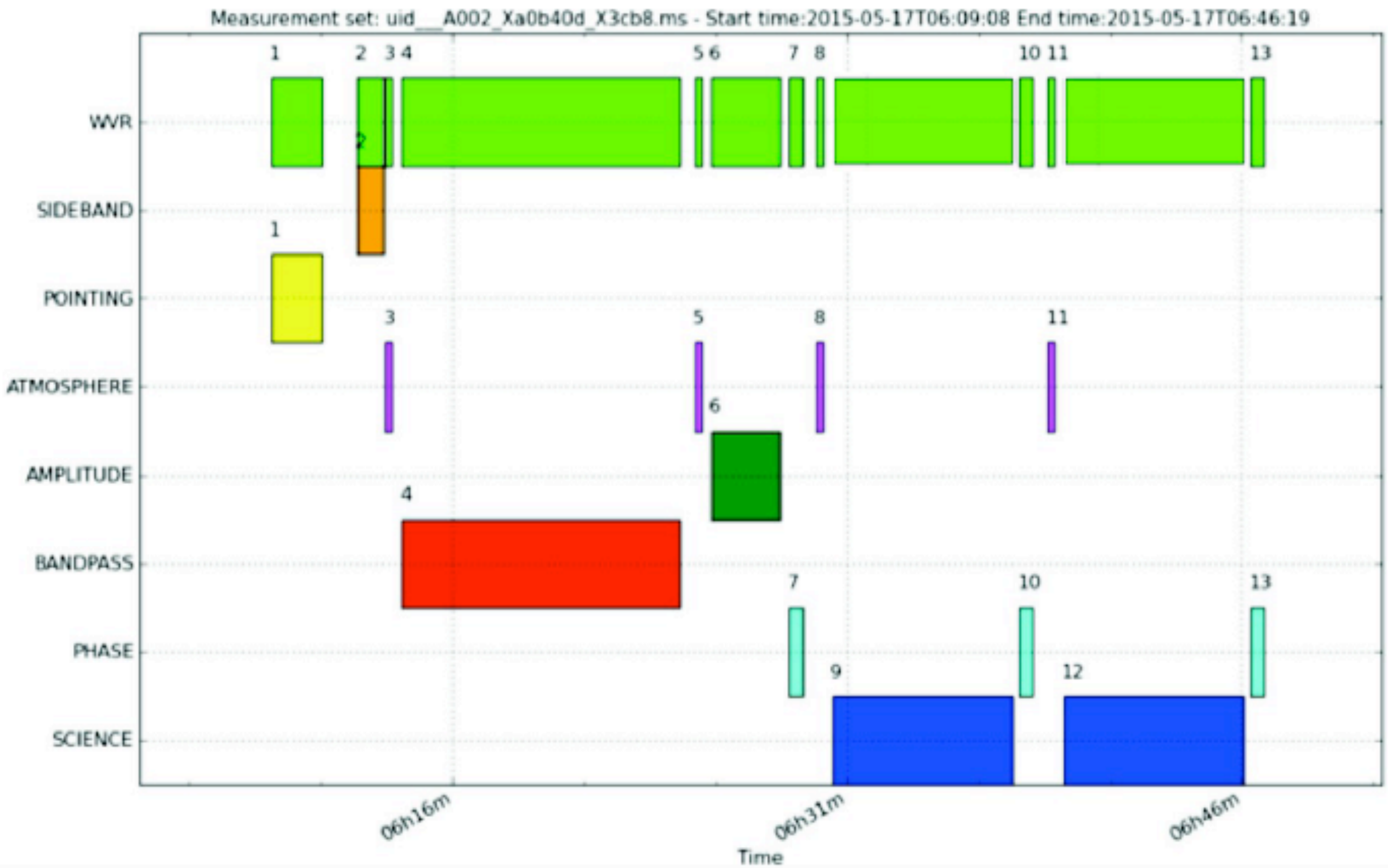
Φ vs t: Troposphere

% corr → K: Tsys

K→ Jy

A, Φ vs ν

Φ vs t: Troposphere



Peculiarities in the mm

- No external human interferences in the data
- No ionospheric effect

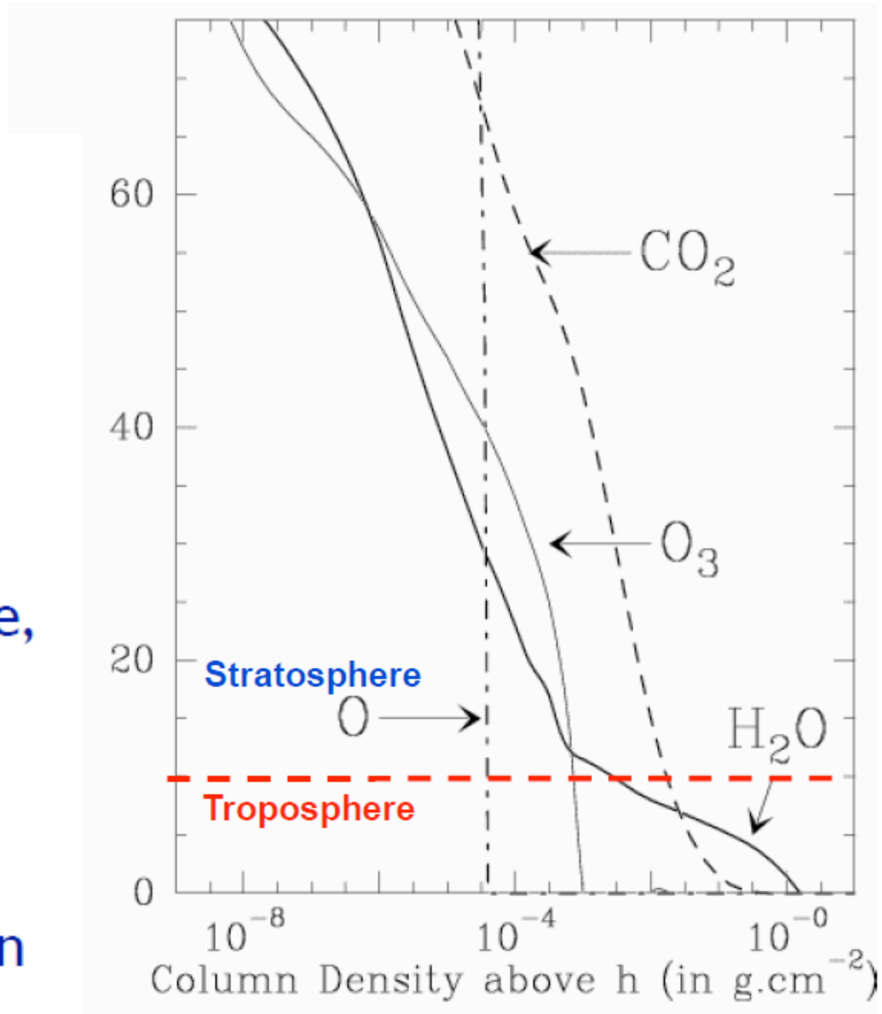
- Troposphere and Tsys are the peculiarities at mm
- (amplitude calibration was for the lack of good calibrators)

Calibration in interferometry: Troposphere @ mm

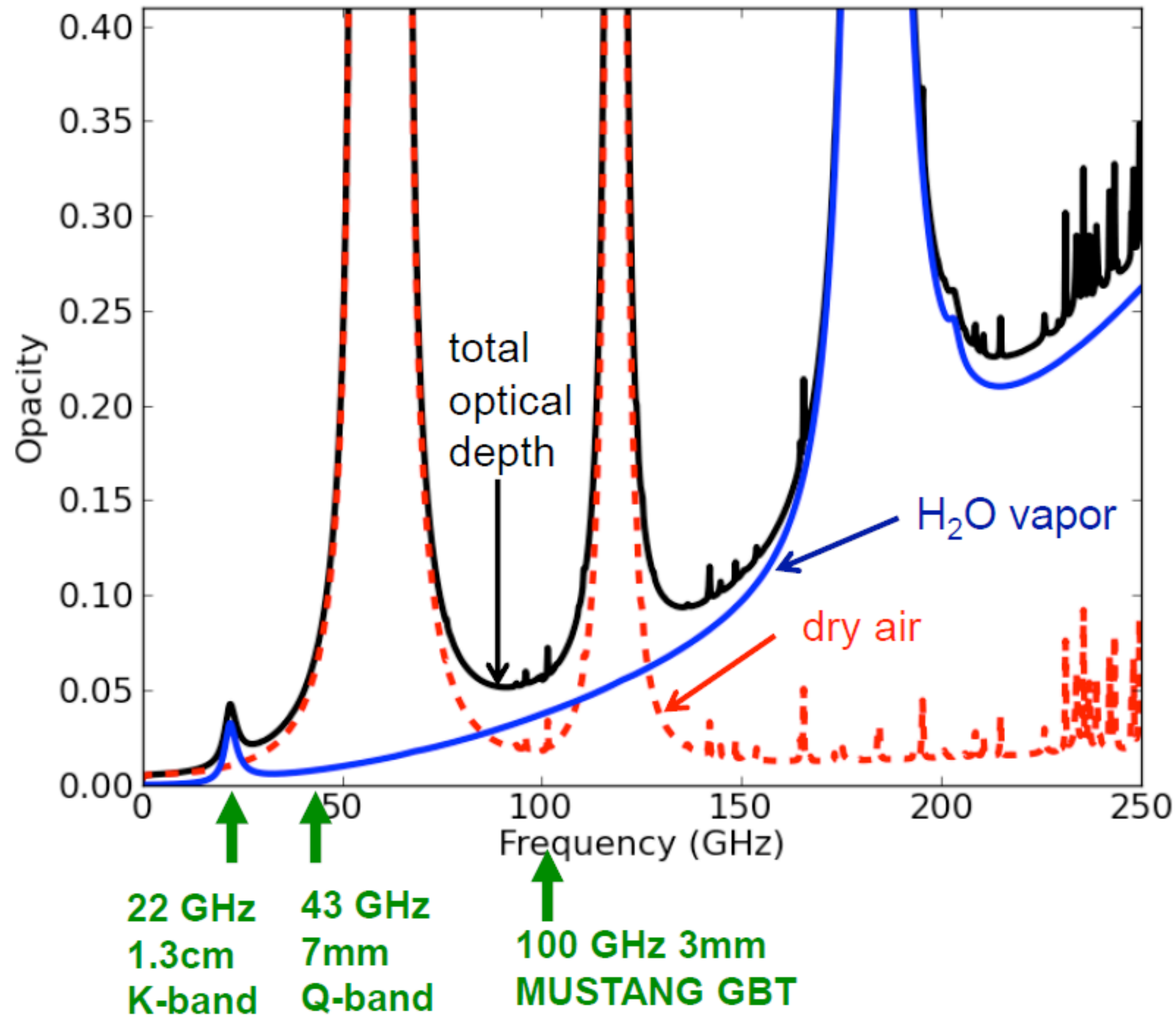
The role of the troposphere

- H_2O (mostly vapor)
- "Hydrosols" (water droplets in clouds and fog)
- "Dry" constituents: O_2, O_3, CO_2, Ne, He, Ar, Kr, CH_4, N_2, H_2
- clouds & convection = time variation

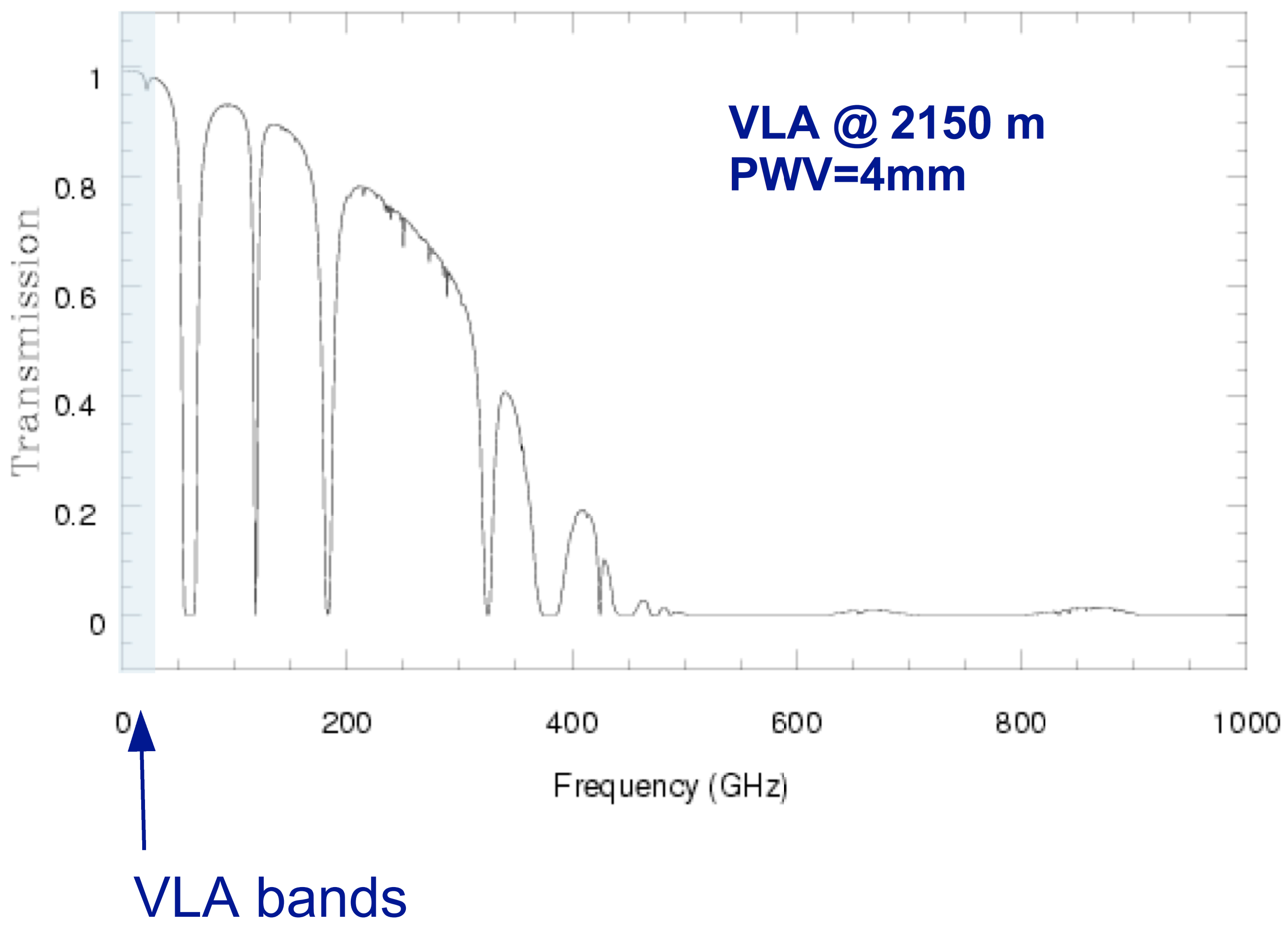
Column density as function of altitude



Optical depth as function of frequency

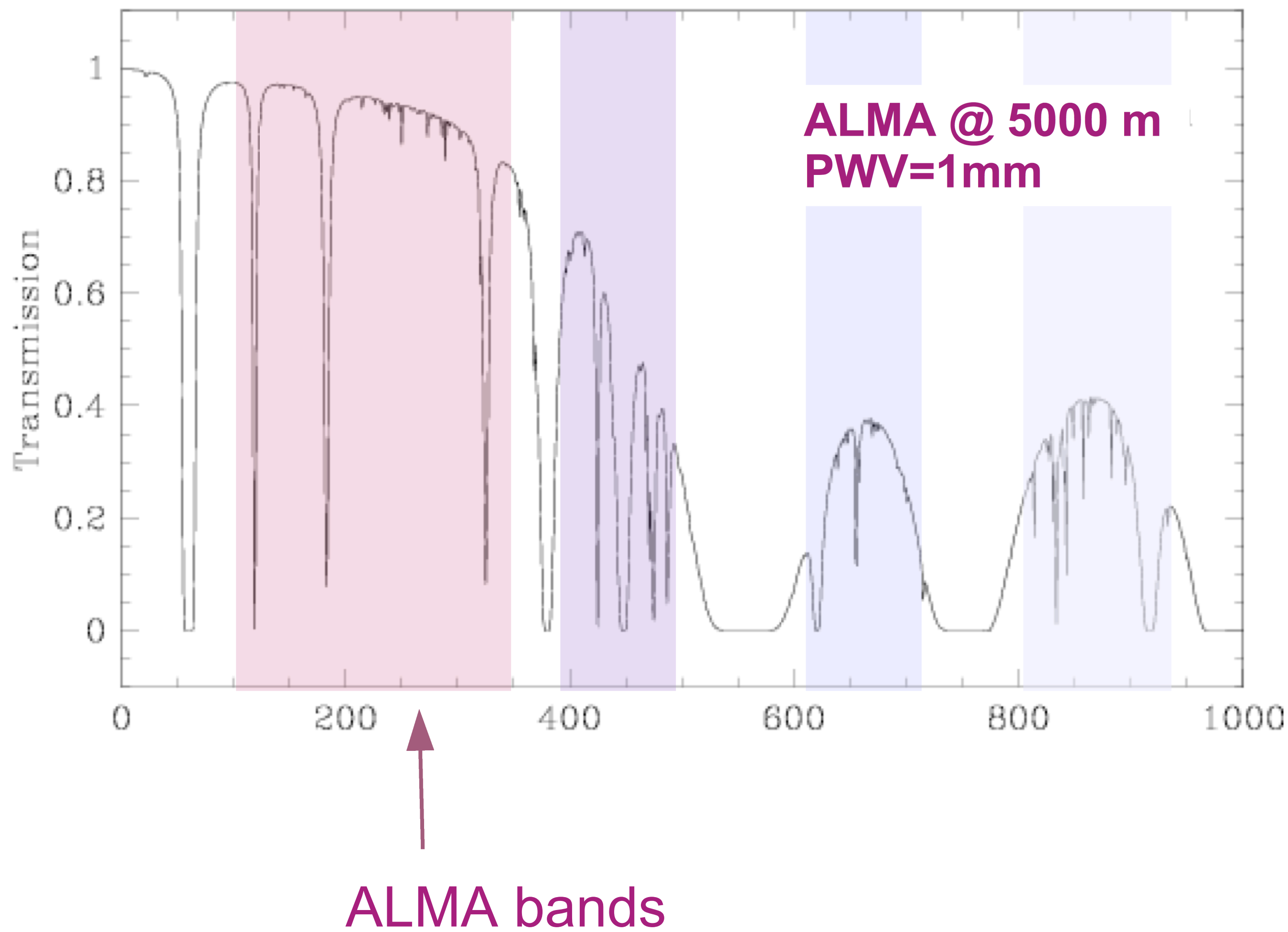


Tropospheric opacity depends on altitude



Atmospheric transmission not a problem @ λ > cm

Tropospheric opacity depends on altitude → importance of dry site



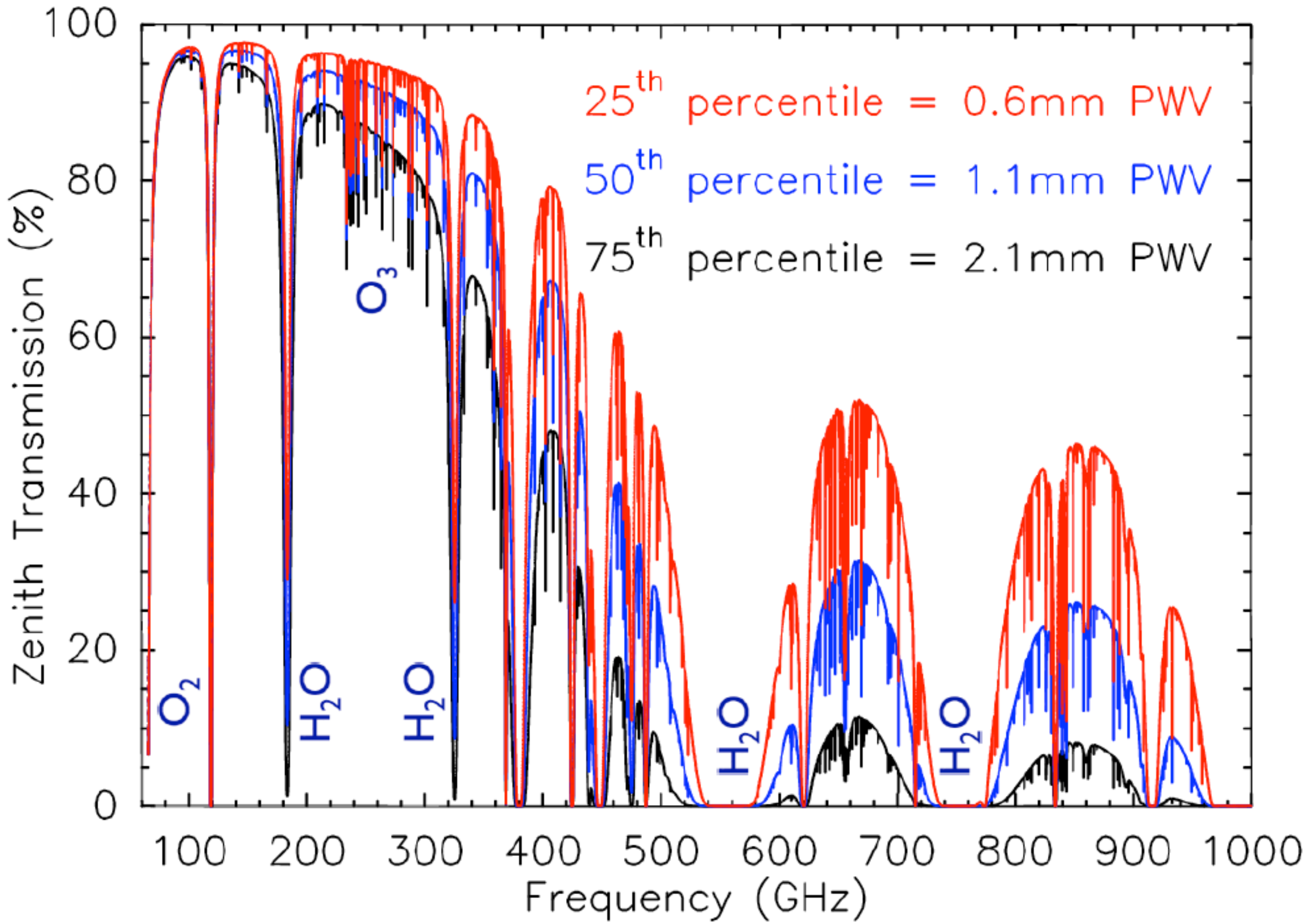
Difference due to the scale height of water vapor

PWV precipitable water vapor

PWV= Precipitable Water Vapour

→ weather conditions dependence

→ dynamic schedule important at high frequencies



Calibration in interferometry: Troposphere @ mm

Effects mainly on Φ vs t due to two troposphere components

Water Vapor

PWV

corrected thanks **WVR calibration**

$$\varphi_e \approx \frac{12.6\pi}{\lambda} \cdot pwv$$

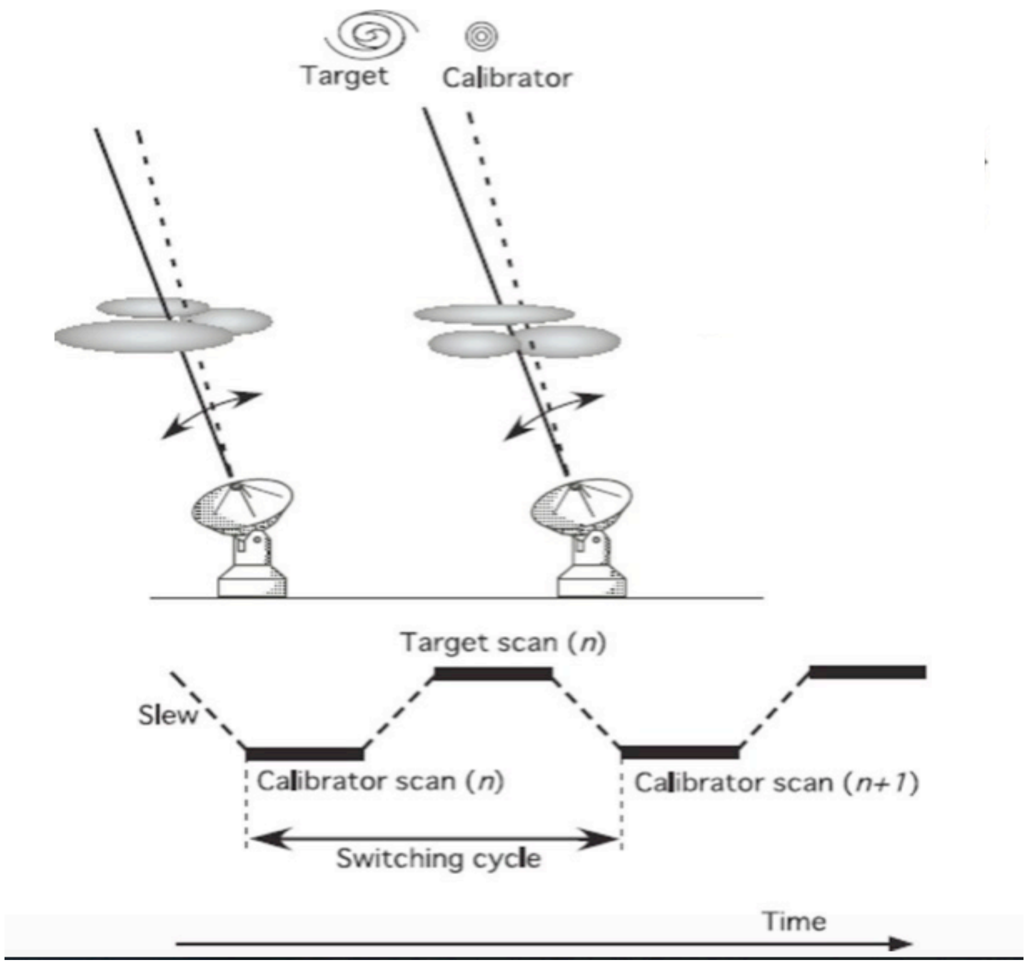
Dry components

Such as O_2, O_3, CO_2

Corrected through **Phase referencing**

Not relevant at the cm

Calibration in interferometry: Dry components @ mm



Phase referencing

Calibrator + target + calibrator

switching cycle?
Angular distance calibrator – target?
Calibrator flux?
Calibrator positional accuracy?

Depend on array site, N antennas, b(max), frequency
→ telescope commissioning activity

Calibration in interferometry: PWV @ mm

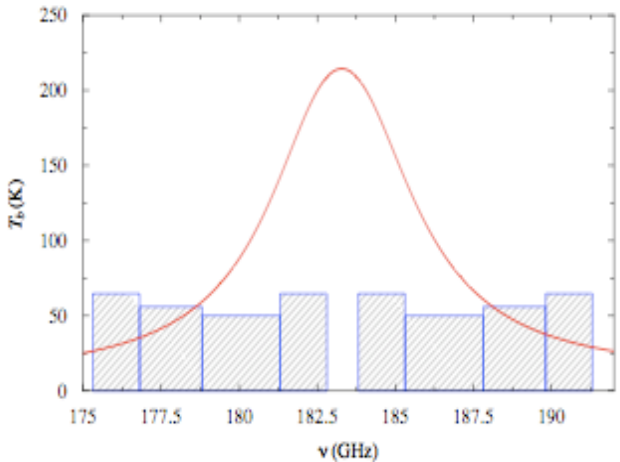
WVR correction

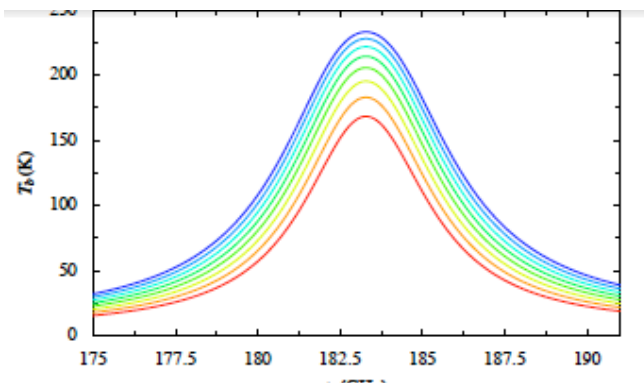
Each ALMA 12 m antenna has a water **vapour radiometer**

Four "channels" flanking the peak of the 183 GHz water line

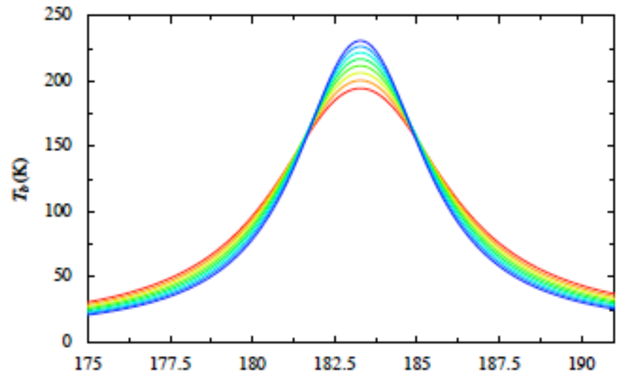
Data taken every second

Convert 183 GHZ brightness to PWV (wvrgcal):
model PWV, temperature and pressure
compare to the observed "spectrum"
compute the correction

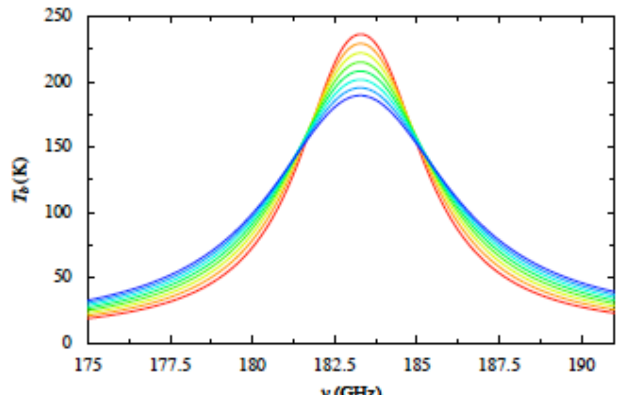




PWV from 0.6 to 1.3 mm



Temperature 230-300 K



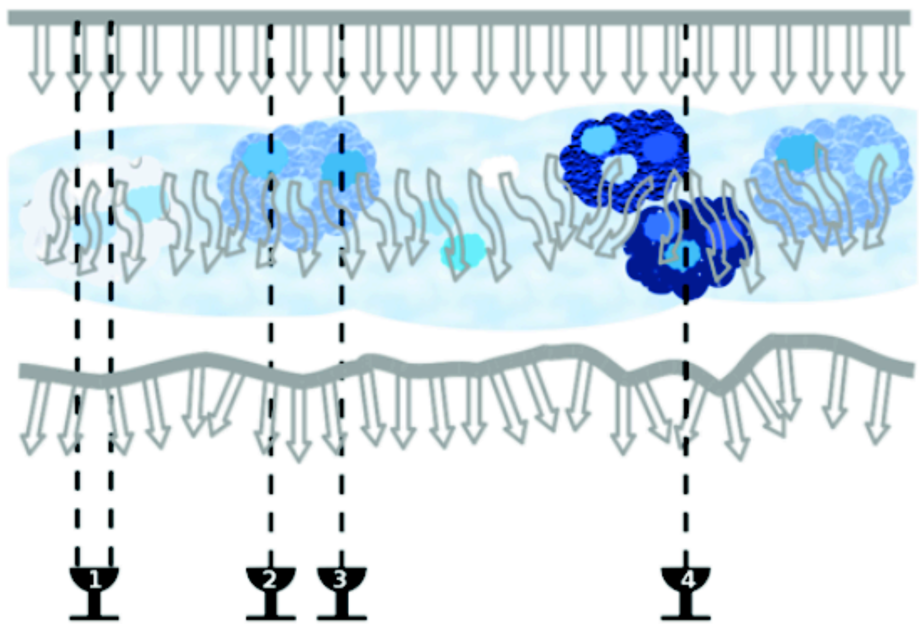
Pressure 400-750 mBar

Calibration in interferometry: troposphere @ mm

PWV and dry components produce two types of effects on Φ due to their

Short time variations (< minute)

Long time variations (min - hour)



Patches of air with different pwv and dry components affect the incoming wave front differently.

Antenna 1, 2, 3 see slightly different disturbances

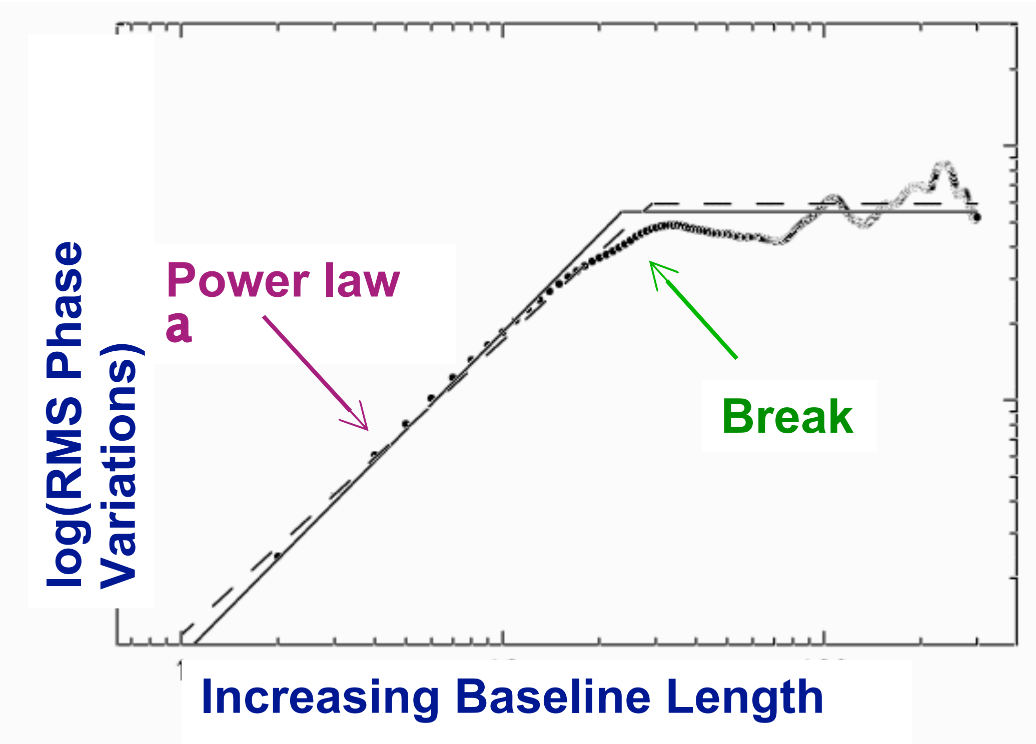
Sky above antenna 4 varies independently

Calibration in interferometry: troposphere @ mm

Mainly effect on phase

Short time variations (< minute)

Spatial structure function



Phase noise

Kolmogorov turbulence theory

$$\varphi_{rms} = \frac{K\, b^{\alpha}}{\lambda}$$

b=baseline length (km)
a = 1/3 to 5/6 (thin or thick atmosphere)
λ = wavelength (mm)
K constant (~100 for ALMA)

The break is typically @ baseline lengths few hundred meters to few km (scale of the turbulent layers)

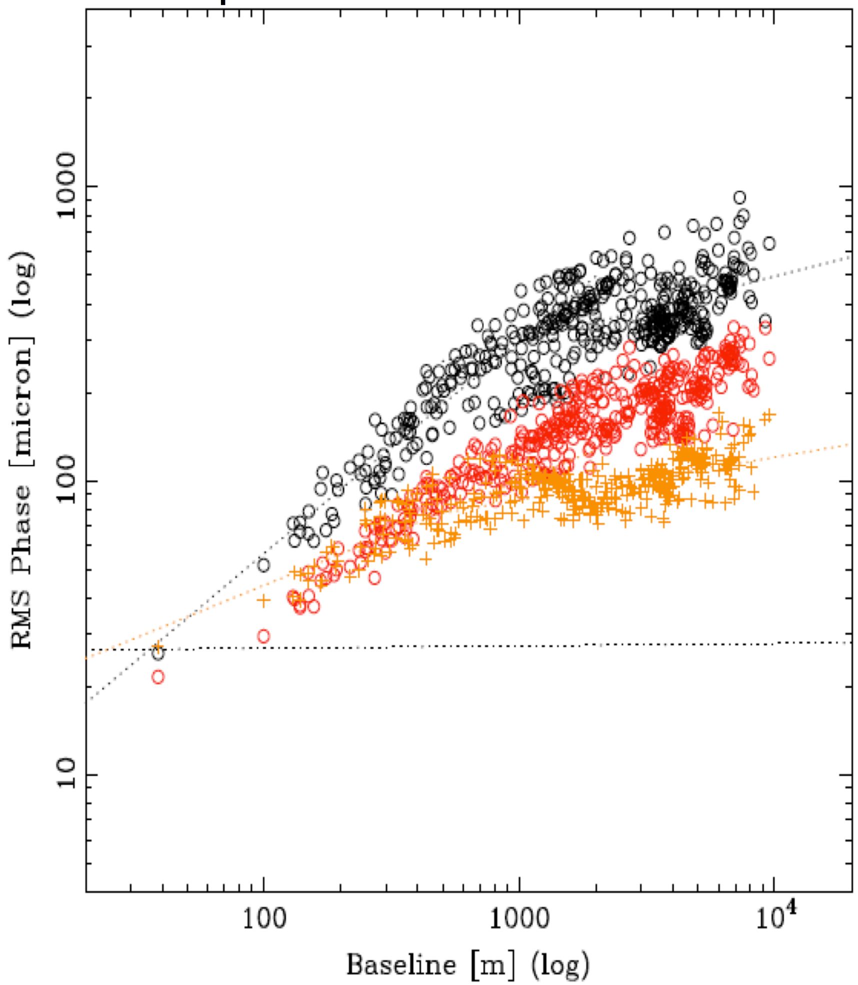
Break and maximum are weather and wavelength dependent

Calibration in interferometry: troposphere @ mm

Mainly effect on phase, but also **amplitude**

Short time variations
(< minute)

Spatial structure function



Decorrelation issue: we lose integrated flux because visibility vectors partly cancel out

$$\langle V \rangle = V_o \langle e^{i\varphi} \rangle = V_o e^{-(\varphi_{rms}^2)/2}$$

No WVR and phase ref correction

WVR correction

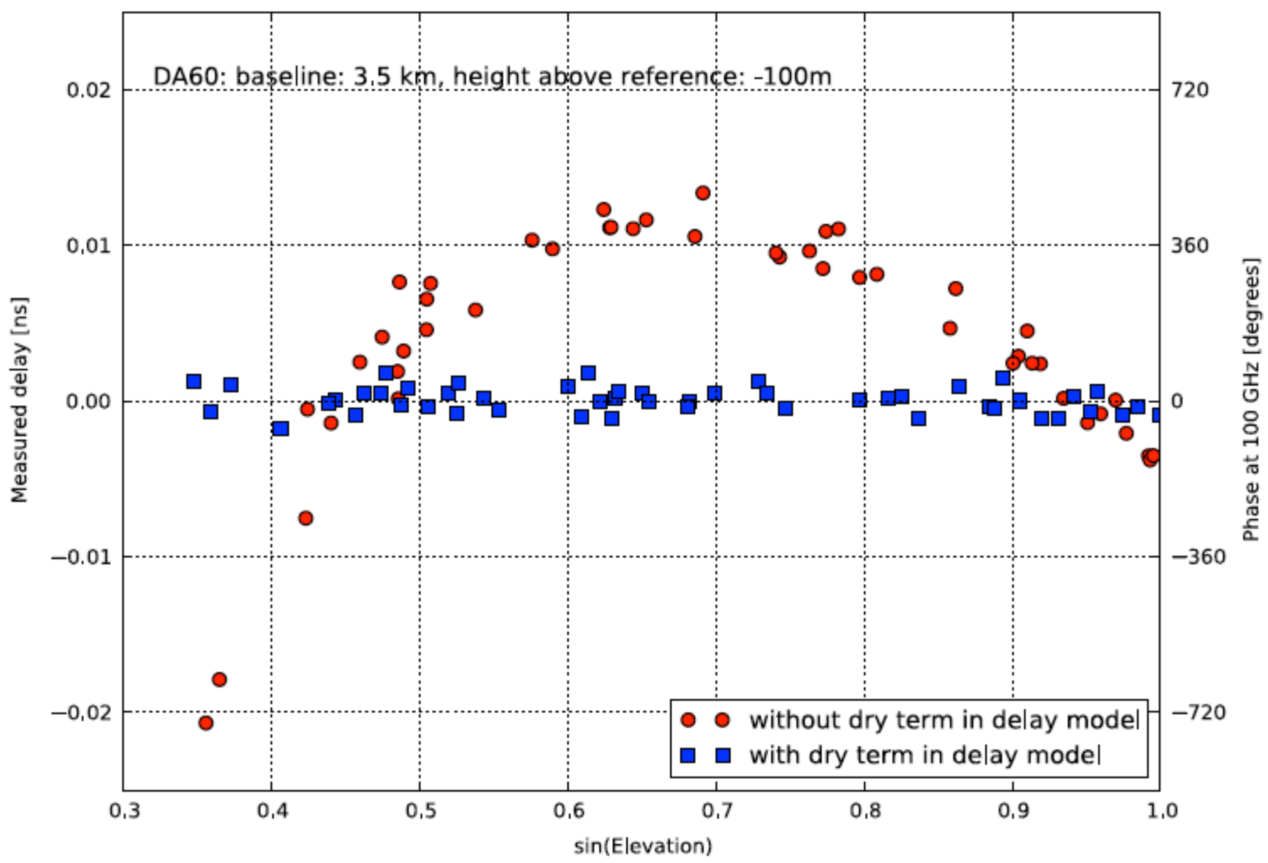
WVR + phase ref corrections

General guideline for accurate images:
$\Phi_{rms} < 30$ deg $\rightarrow$ $\langle V \rangle = 0.90\, V_0$

Calibration in interferometry: troposphere @ mm

Mainly effect on phase

Delay vs sin(elevation)



Long time variations
(min - hour)

Only WVR calibration

WVR + phase ref calibration

Calibration in interferometry:
Tsys @ mm

Assuming correlated data in units of % correlation multiplication by
Tsys will change the unit to Kelvin

System noise temperature

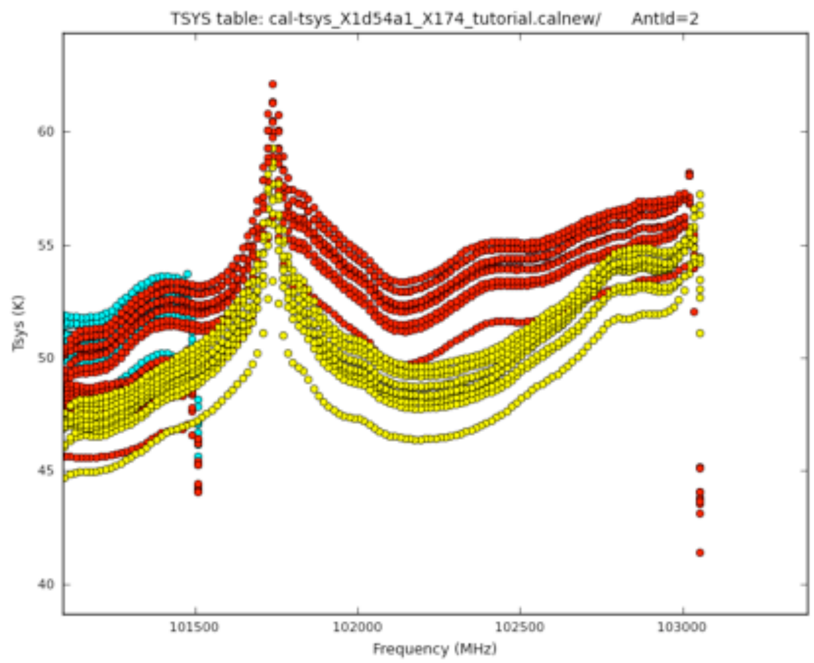
$$T_{sys} \sim T_{atm}\,(e^{t}-1) + T_{rx}\,e^{t}$$

At lower frequencies T_{rx} is dominant

At higher frequencies (mm/submm), the noise associated with the atmosphere T_{atm} is dominant, and acts like a blackbody emitter, attenuating the astronomical signal

ALMA front end are equipped with an **Amplitude Calibration Device** (ACD) to measure T_{sys}

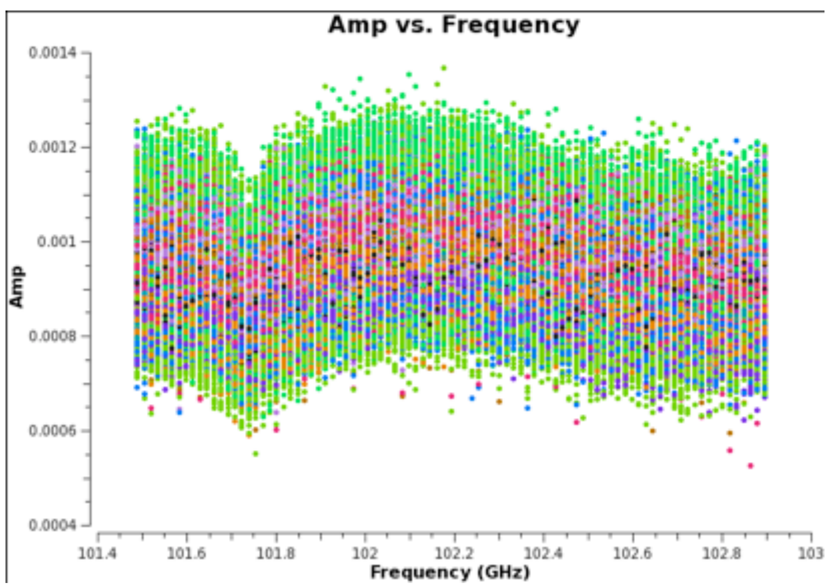
Every scan could have a Tsys measurement, but <400 GHz relatively constant ~10min.
Tsys spectra are applied off-line to the correlated data.



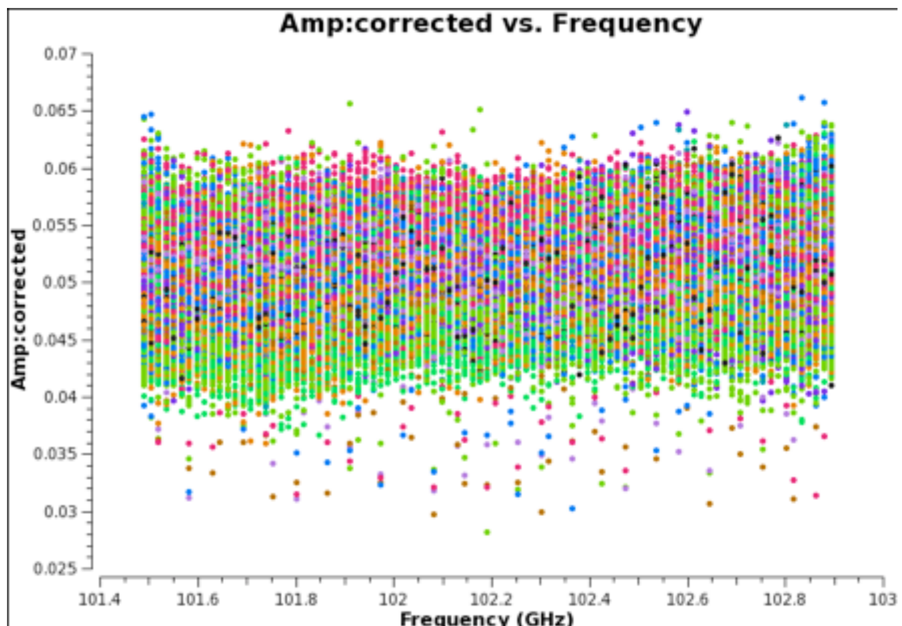
Tsys calibration
corrects for atmosphere opacity
(and fake line absorption in spectra)

Spectral Tsys
band 3 (~100 GHz)

Before



After



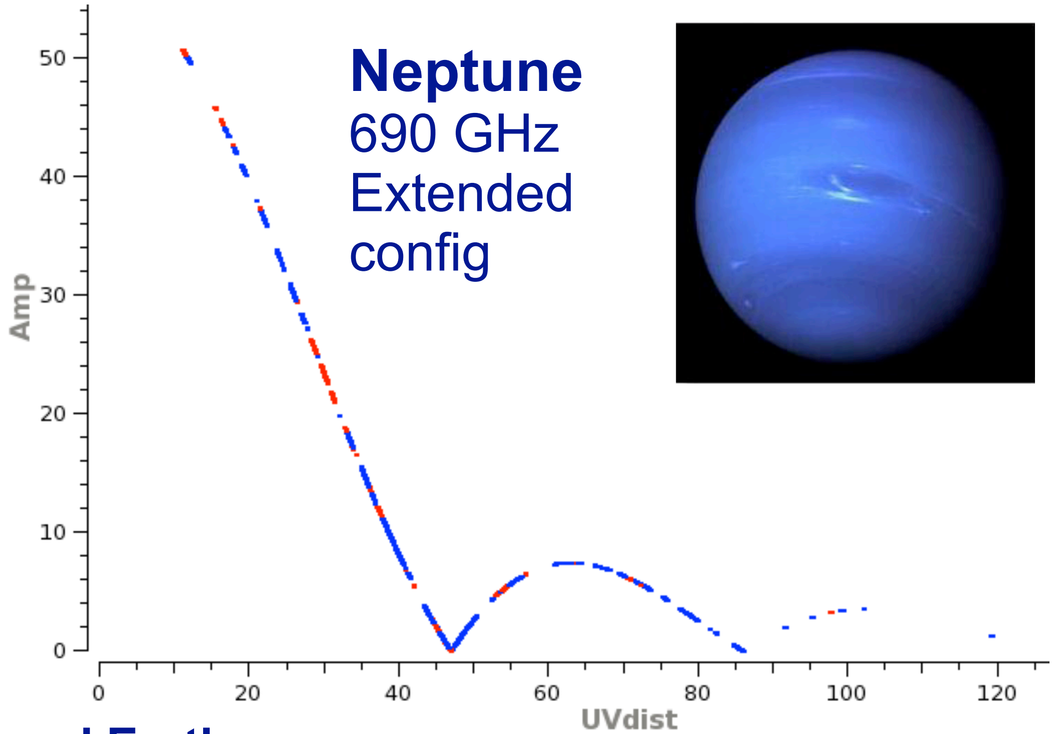
Calibration in interferometry: Flux calibrators @ mm

Tsys changes the unit to Kelvin, **the flux calibration will change Kelvin to Jy**

In early ALMA cycles

- Quasars are strongly time-variable and good models did not exist at high frequencies
- Solar System bodies are used as primary flux calibrators (Neptune, Jovian moons, Titan, Ceres) but with many challenges:
  - **all are resolved on long baselines**
  - **brightness varies with distance from Sun and Earth**
  - **line emission present → need models**

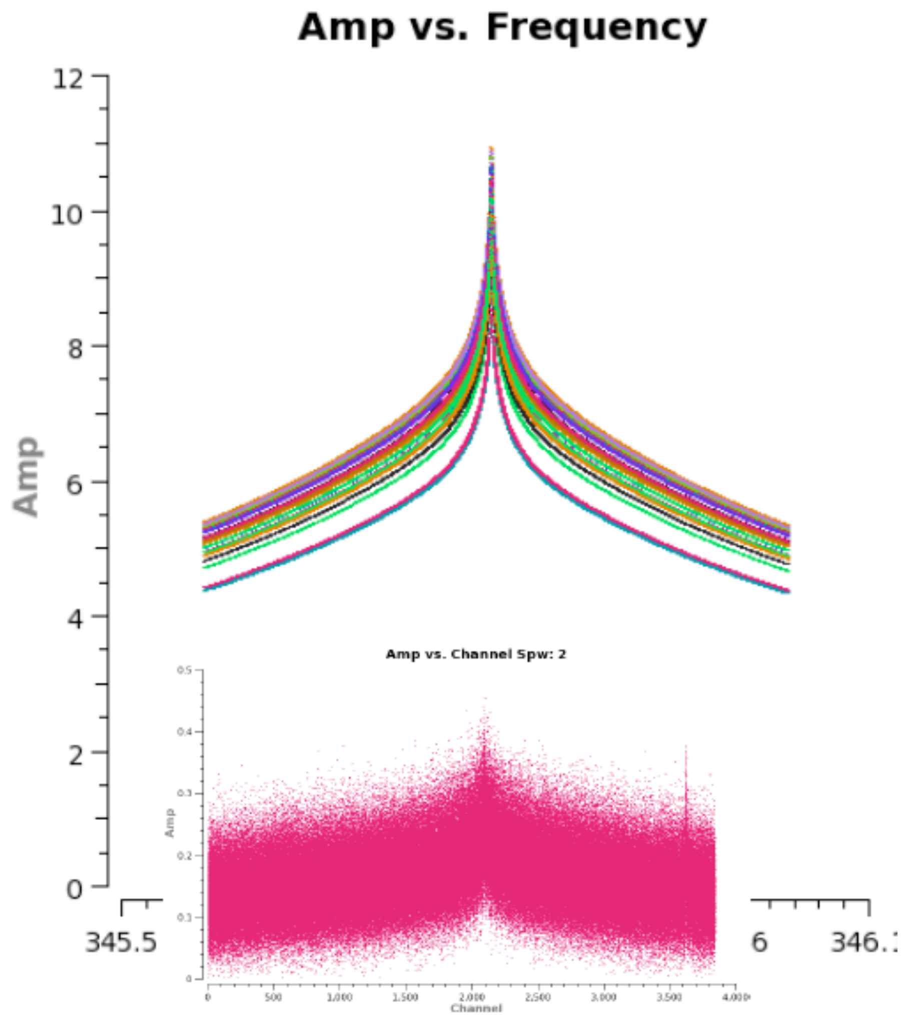
Other possibilities: asteroids, red giant stars...
Monitoring of point-like quasars done!



Neptune
690 GHz
Extended
config

Flux calibrators

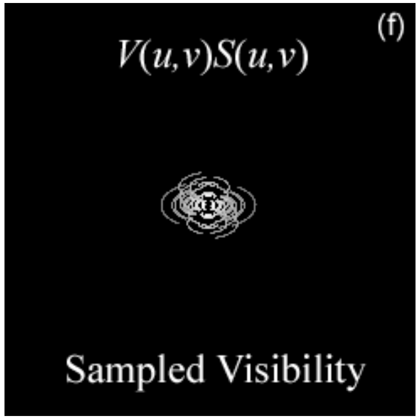
Model spectral lines: CO in Titan



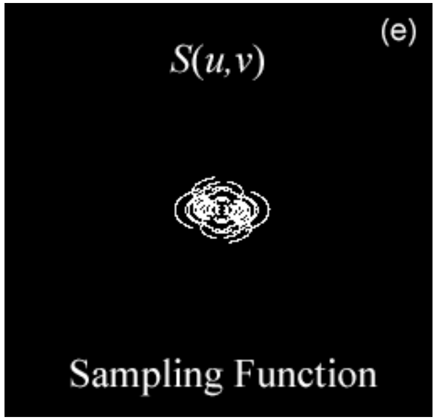
(u,v) sampling vs image fidelity

But: we actually sample the Fourier domain (u,v) at discrete points resulting in an imperfect image

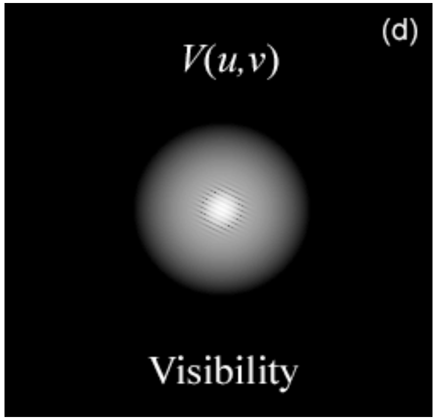
Observed $V_{cal}(u,v)$ = sampling function $S(u,v)$ × Real $V_{true}(u,v)$



=



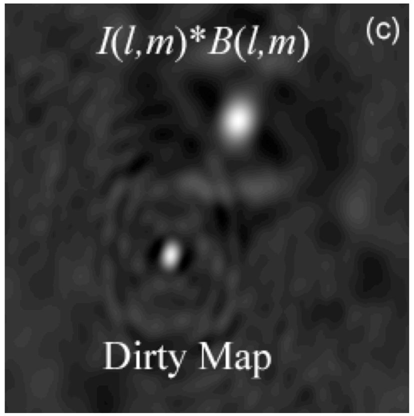
X



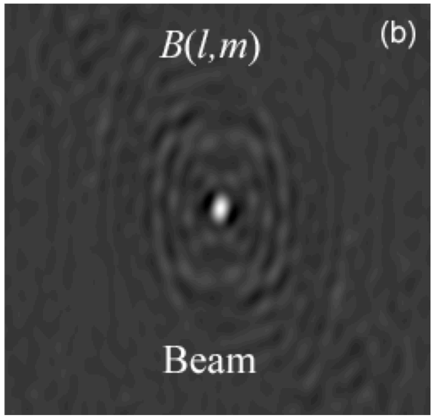
S (u,v) is the sampling function:

- S= 1 at points where V(u,v) are measured,
- S = 0 elsewhere

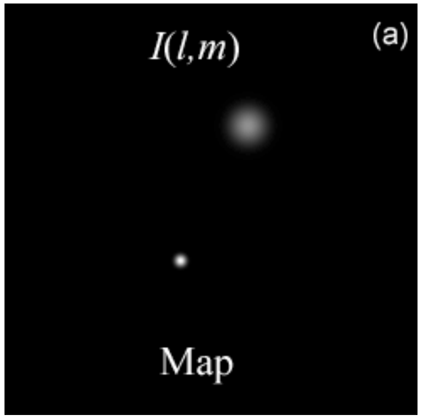
$FT^{-1} V_{cal}(u,v)$ = $FT^{-1} S(u,v)$ × $FT^{-1} V_{true}(u,v)$



=



X



1 point source + Gaussian

$FT^{-1}(S) = B_{dirty}(x,y)$ is the Dirty Beam

Deconvolution process

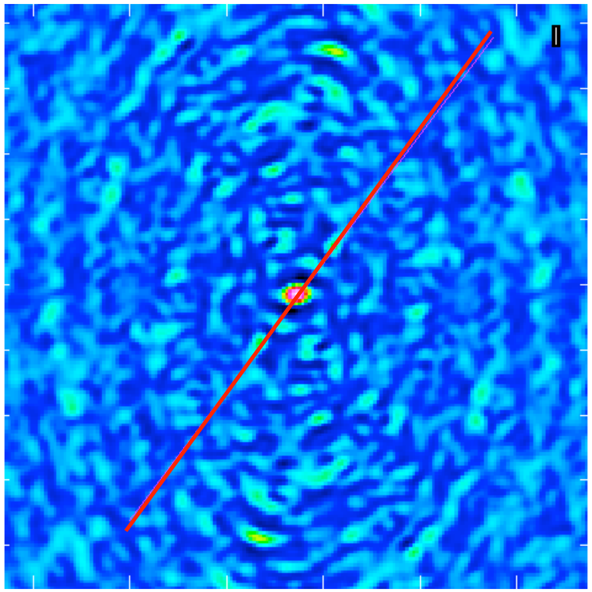
$$I^D(x,y) = B_{dirty}(x,y) \otimes I(x,y)$$

Nyquist sample:
Image size: 2 FOV
Image pixel size: usually 1/4 or 1/5 of the synthesized beam

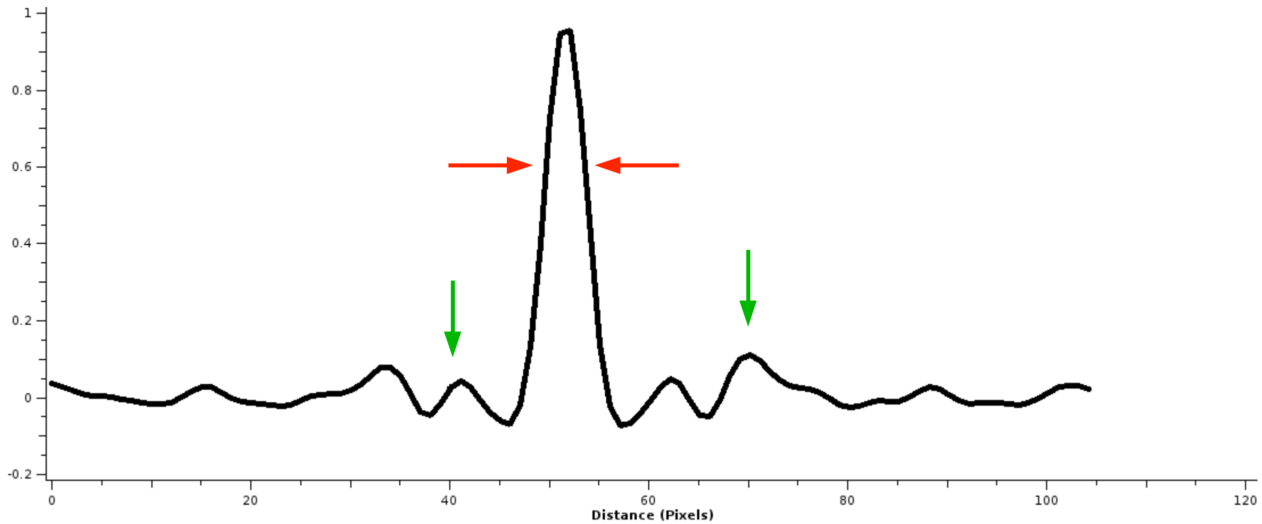
Imperfect reconstruction of the sky

- **Incomplete sampling of uv plane → sidelobes**

$B_{dirty}(x, y)$



- Central maximum has width $1/(u_{max})$ in x and $1/(v_{max})$ in y
- Has ripples (sidelobes) due to gaps in uv coverage



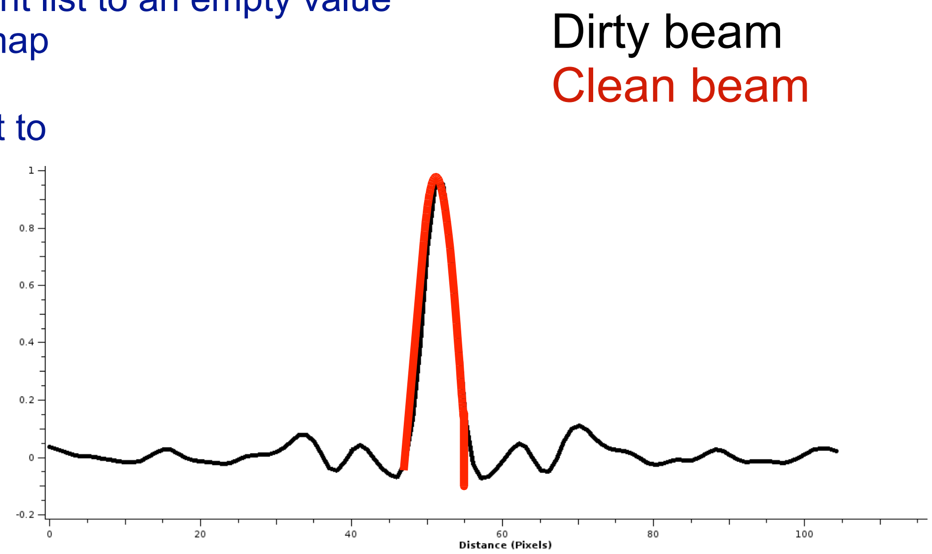
deconvolution → sidelobes removal, e.g. in the clean process
Hogbom 1974, Clark 1980, Cotton-Schwab 1984

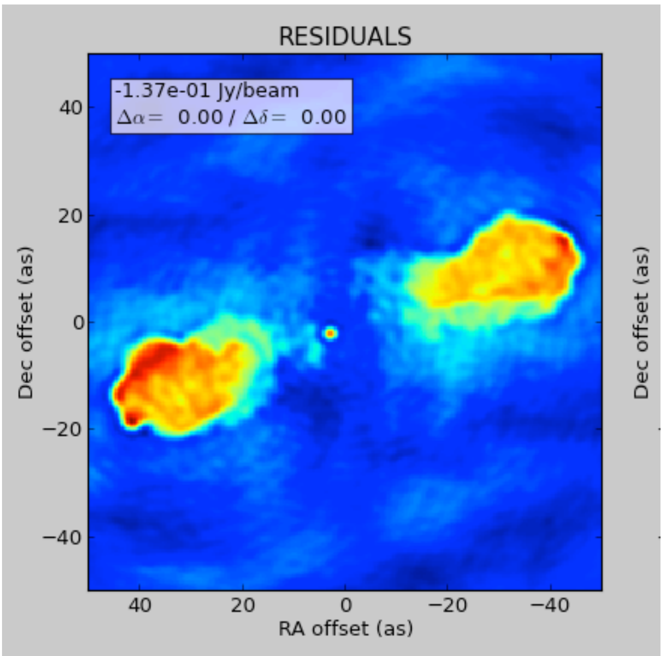
Deconvolution - Classic CLEAN

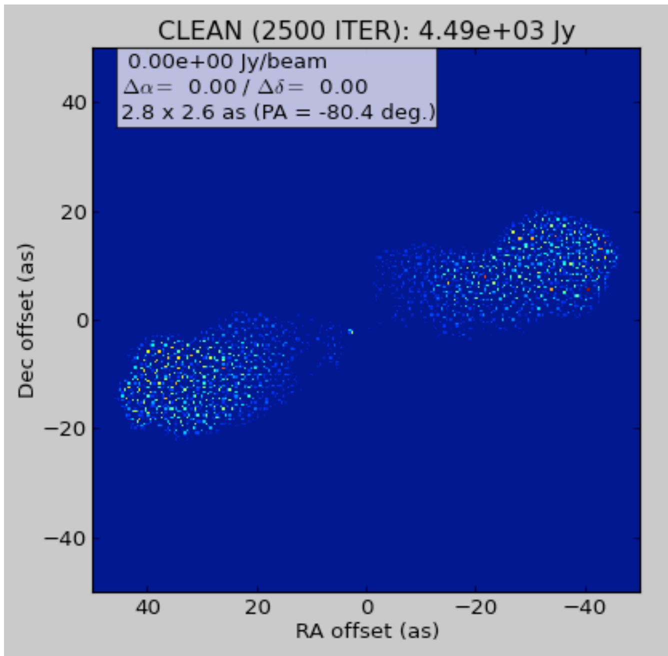
Hogbom 1974, Clark 1980, Cotton-Schwab 1984

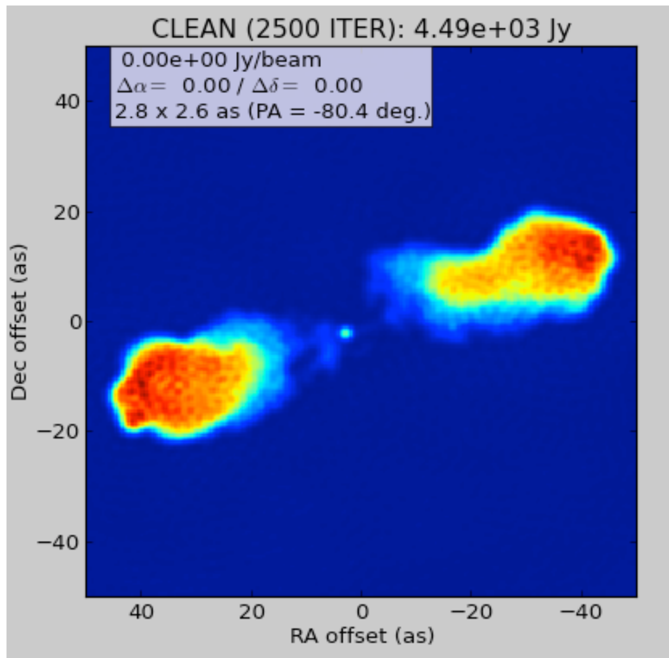
Basic assumption: **each source is a collection of point sources**

- Initializes the residual map to the dirty map and the Clean component list to an empty value
- Identifies the pixel with the peak of intensity **(Imax)** in the residual map and adds to the clean component list a fraction of Imax = g Imax
- **Multiplies the clean component by the dirty beam** and subtract it to the residual
- Iterates until **stopping** criteria are reached
  |Imax| < multiple of the rms (when rms limited),|Imax| < fraction of the brightest source flux (when dynamic range limited)
- **Multiplies the clean components by the clean beam** an elliptical gaussian fitting the central region of the dirty beam
  → restoring







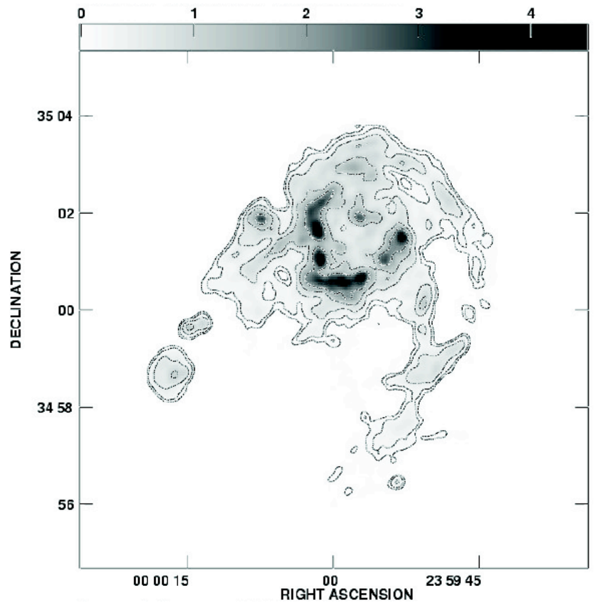


PB corrected images

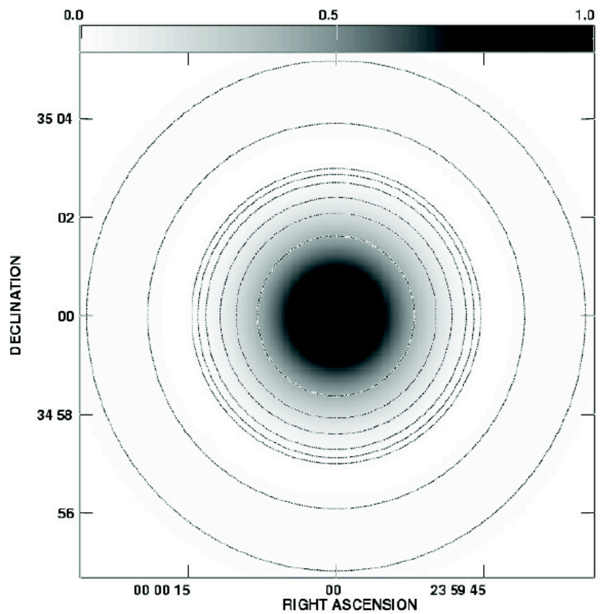
But

Interferometer elements are sensible to direction of arrival of the radiation

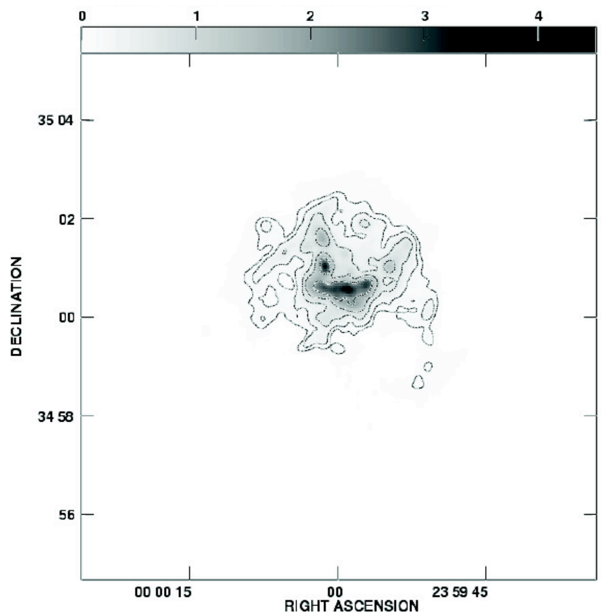
- **Primary beam effect** $\longrightarrow$ $T(x,y) = A(x,y)\ T'(x,y)$



Model of emission
Larger than the PB



PB sensitivity pattern



PB applied
sensitive to center
emission only

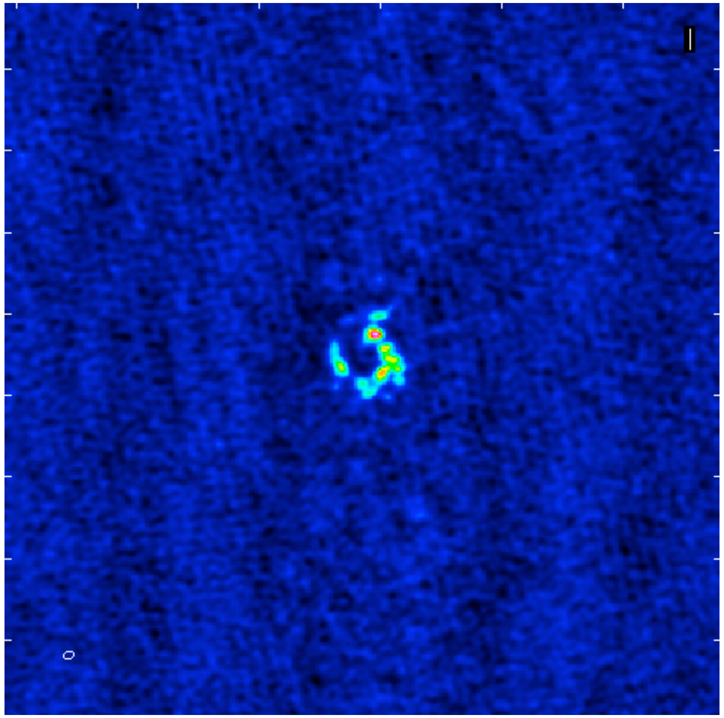
The response of the antennas in the array must be corrected for during imaging to get accurate intensities for source outside the core of the beam.

We need to get $T(x,y) = \int\int V(u,v)e^{-2\pi i(ux+vy)}dudv$

But

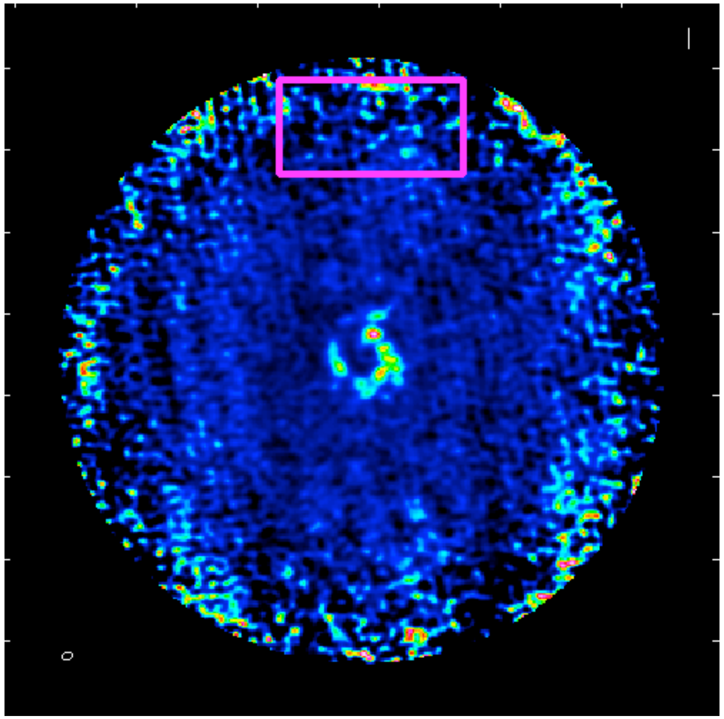
- **Primary beam effect** $\longrightarrow$ $T(x,y) = A(x,y)\ T'(x,y)$

$T(x,y)$



rms 8e-4

$T'(x,y)$



rms 3e-3

ALMA provides images that are primary beam corrected!

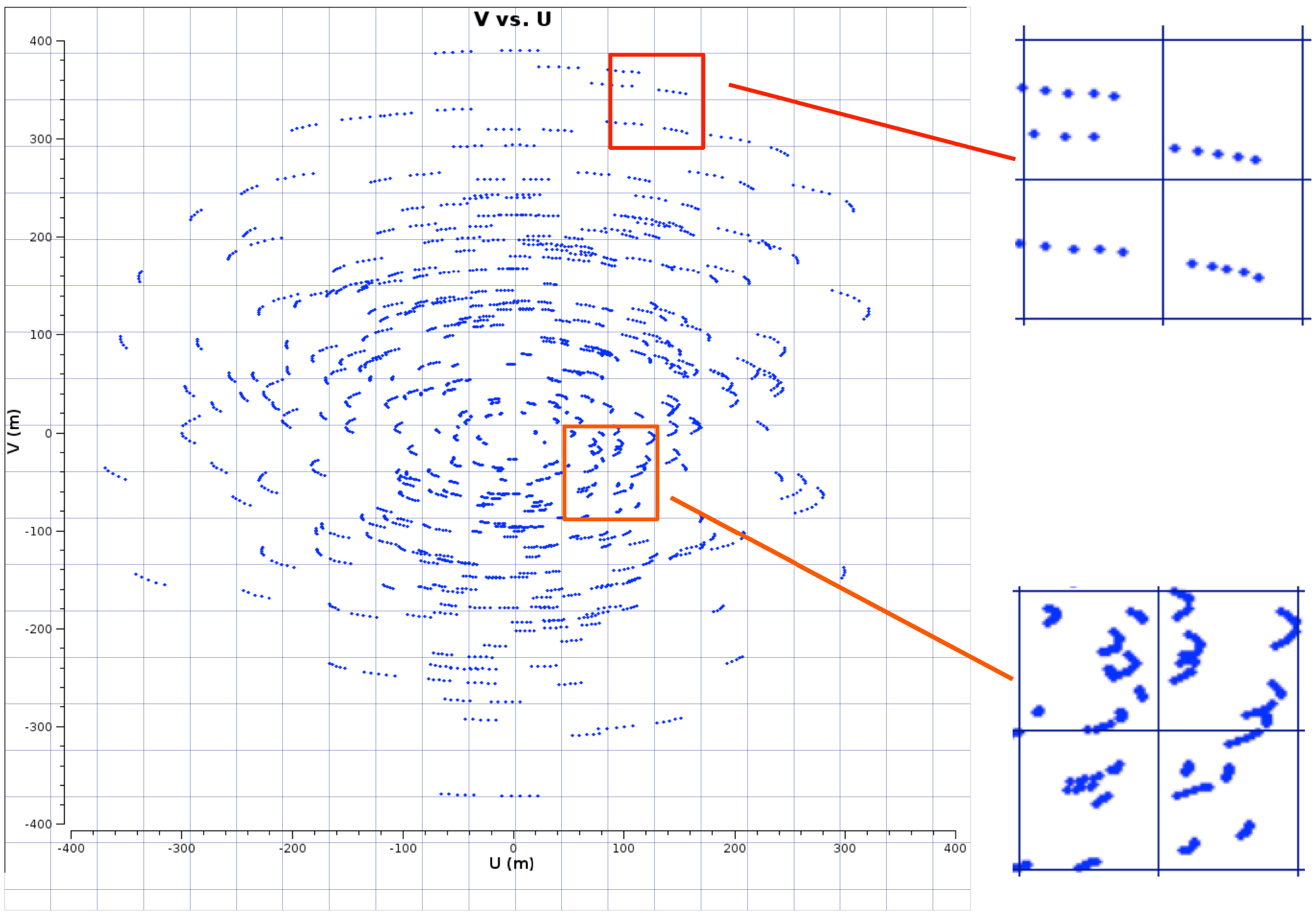
Different weighted images

But

…. some uv ranges are sampled more than others

- **Gridded visibilities are →** $V(u,v) = W(u,v)\ V'(u,v)$

Tipically, short spacing
are sampled more than long

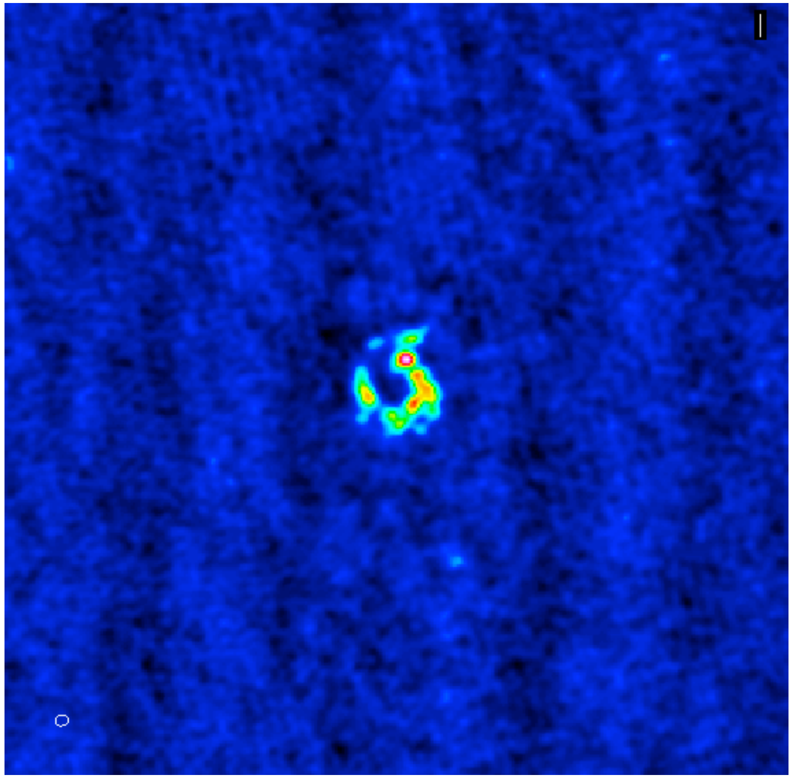


Different weighting **W(u,v)** :

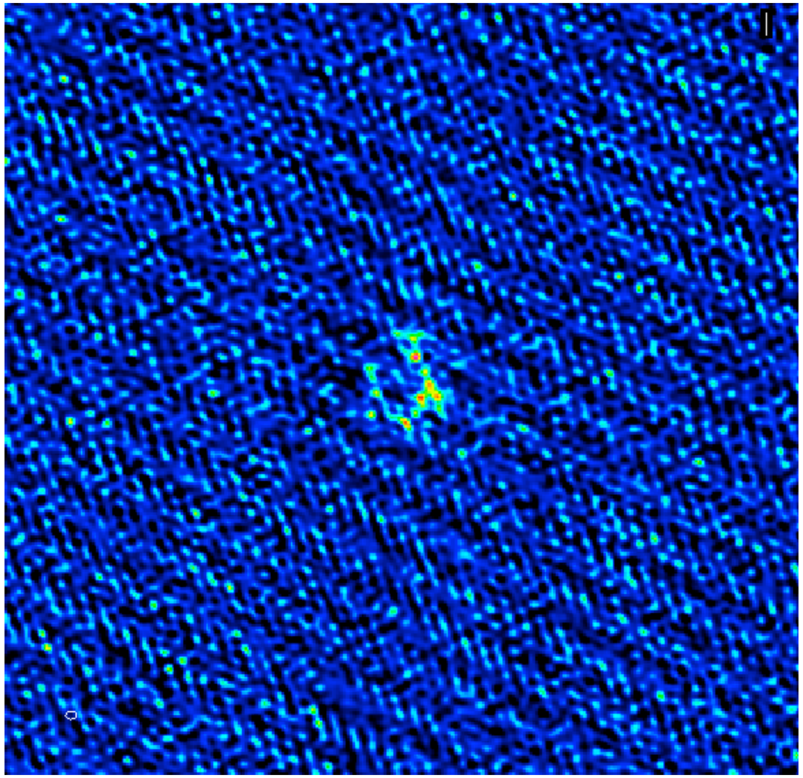
- Uniform:
  long baseline, < ϑres
- Natural:
  short baseline, < rms
- **Briggs:** intermediate
  → provided in the ALMA archive images

Weighting effects on the image

Natural
res = 0.29" x 0.23"
rms = 0.8 mJy/beam



Uniform
res = 0.24"x0.17"
rms = 3 mJy/beam

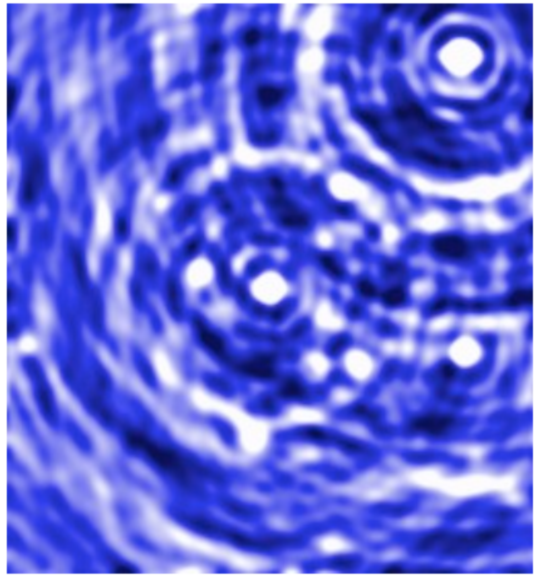


Note: Other different final images are possible (uv tapering, uv range selection, multi-scale, wide field) depending on the science case

High dynamic range vs image fidelity

- Bright sources in the field of view introduce strong sidelobes which affect the rms in the clean image → images could be high dynamic range limited
- **Imaging Dynamic Range (IDR)= Peak flux / RMS**
- ALMA guarantees:
  - IDR < 100 (1000 for spectral line images) for Bands < 9
  - IDR < 50 (500 for spectral line images) for B9 and B10

  → Higher IDR must be justified and require selfcal!

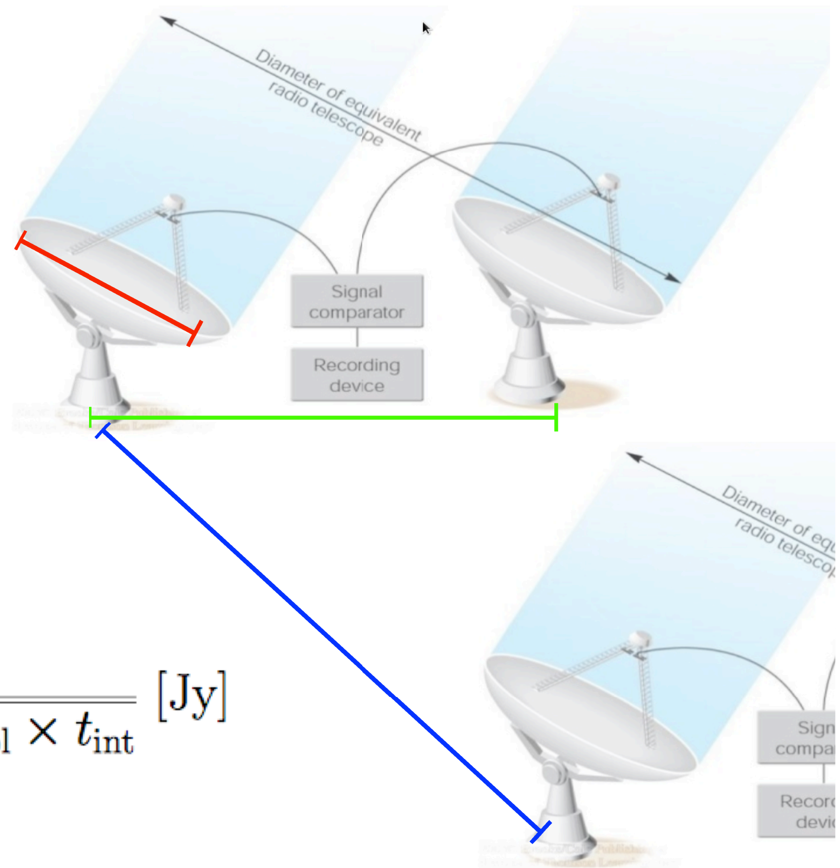


- **Selfcal** to improve high dynamic range limited images (but not only):

Selfcal idea:

- **Calibration using external calibrators in not perfect** interpolated from different time, different sky directions from source
- **Basic idea of self-calibration:** objects with enough S/N can be used to calibrate themselves to obtain a more accurate image.
- It works because
  - at each time the number of baselines is much larger than the number of antennas (of complex gains)
  - source structure can be represented by a small number of parameters
  - **It is dangerous in case of arrays with a small number of antennas and complex sources**
  - **Rule of thumb: it is worth using it if S/N > 20** (for an array of 25 antennas) Amplitude self-cal is only effective if >90% of the flux density is in the image after phase self-cals

Interferometry: summary in practice



- **Field of View**
  → single pointing or mosaic?

$$FOV \propto \frac{\lambda}{D}$$

- **Angular resolution**
  → array configuration?

$$\theta_{res} \approx \frac{\lambda}{B_{max}}$$

- **Maximum recoverable scale**
  → total power observations?

$$\theta_{MRS} \approx \frac{\lambda}{B_{min}}$$

- **Sensitivity**
  → Δν of spectral obs?

$$\sigma_S \approx \frac{2\,k\,T_{sys}}{A_{eff}\,\sqrt{n(n-1)} \times \Delta\nu \times \eta_{pol} \times t_{int}}\ [\mathrm{Jy}]$$

- Interferometer measures **V(u,v) = FT (T(x,y))**
  → Calibration of V(u,v) (=A, Φ vs ν t), imaging is FT^{-1} (V(u,v))

- In the mm, the **troposphere effects** on Φ (due to PWV and dry components) are dominant and increase with ν and b → high ν and long b observations more difficult.
  (ALMA requires also Tsys calibration, initial problems also with flux calibration)

- **Image fidelity** strongly depends on the (u,v) coverage and source dynamic range
  → best (u,v) coverage, selfcal needed?

- Different weighted images could be produced depending to the science case