Ideal Visibilities: $\boldsymbol{V^{true}}$

$$V_{XX} = \mathcal{I} + \mathcal{Q}$$
$$V_{XY} = \mathcal{U} + i\mathcal{V}$$
$$V_{YX} = \mathcal{U} - i\mathcal{V}$$
$$V_{YY} = \mathcal{I} - \mathcal{Q}$$

Stokes visibilities

$$I = V_{XX}+V_{YY} /2$$
$$Q= V_{XX}-V_{YY}/2$$
$$U= V_{XY}+V_{YX}/2$$
$$V=V_{XY}-V_{YX}/2i$$

Parallactic Angle: $\boldsymbol{P\ V^{true}}$

$$V_{XX} = \mathcal{I} + (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} + \mathcal{Q}_\psi$$

$$V_{XY} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) + i\mathcal{V} = \mathcal{U}_\psi + i\mathcal{V}$$

$$V_{YX} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) - i\mathcal{V} = \mathcal{U}_\psi - i\mathcal{V}$$

$$V_{YY} = \mathcal{I} - (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} - \mathcal{Q}_\psi$$

$$Q_\psi = Q\cos 2\psi + U\sin 2\psi$$

$$U_\psi = -Q\sin 2\psi + U\cos 2\psi$$

$$\psi = \psi(t)$$

D in the Linear Basis **V= DPV**$^{\mathbf{true}}$:

- Linearized, sorted:

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{U}_\psi+i\mathcal{V})d_{Xj}^* + d_{Xi}(\mathcal{U}_\psi-i\mathcal{V})$$

$$V_{XY} = (\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{I}+\mathcal{Q}_\psi)d_{Yj}^* + d_{Xi}(\mathcal{I}-\mathcal{Q}_\psi)$$

$$V_{YX} = (\mathcal{U}_\psi-i\mathcal{V}) + d_{Yi}(\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{I}-\mathcal{Q}_\psi)d_{Xj}^*$$

$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + d_{Yi}(\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{U}_\psi-i\mathcal{V})d_{Yj}^*$$

D in the Linear Basis **V= DPV^true^**:

- Linearized, sorted, $d\mathcal{V} \sim 0$, regrouped Stokes

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Xj}^*+d_{Xi})$$

$$V_{XY} = (\mathcal{U}_\psi+i\mathcal{V}) + \mathcal{I}\,(d_{Yj}^*+d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}^*-d_{Xi})$$

$$V_{YX} = (\mathcal{U}_\psi-i\mathcal{V}) + \mathcal{I}\,(d_{Yi}+d_{Xj}^*) + \mathcal{Q}_\psi\,(d_{Yi}-d_{Xj}^*)$$

$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Yi}+d_{Yj}^*)$$

D in the Linear Basis **V= DPV^{true}**:

- Linearized, sorted, $d\mathcal{V} \sim 0$, regrouped Stokes

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Xj}^* + d_{Xi})$$

$$V_{XY} = (\mathcal{U}_\psi + i\mathcal{V}) + \mathcal{I}\,(d_{Yj}^* + d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}^* - d_{Xi})$$

$$V_{YX} = (\mathcal{U}_\psi - i\mathcal{V}) + \mathcal{I}\,(d_{Yi} + d_{Xj}^*) + \mathcal{Q}_\psi\,(d_{Yi} - d_{Xj}^*)$$

$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Yi} + d_{Yj}^*)$$

Complex offset in the cross-hands constant in time

In all correlations a d-scaled time-dependent source linear polarization

CASA/ALMA Polarization Calibration Model

- Calibration Model:

$$V^{obs} = K^{crs}\, B^r\, G^r\, D^r\, X^r\, P\, V^{mod}$$

- Basic Solve sequence:
  - Normal bandpass (B^r) and gain (G^r) (parallel-hands):

$$V^{obs} = \underline{B^r}\, V^{\mathcal{I}}$$

$$(B^{r\,-1}\, V^{obs}) = \underline{G^r}\, V^{\mathcal{I}} \quad \text{(+estimate } \mathcal{QU}\text{)}$$

  - Cross-hand delay (K^{crs}), phase (X^r), and $\mathcal{Q},\mathcal{U}$ (cross-hands):

$$V^{obs} = \underline{K^{crs}}\,(B\, G\, P\, V^{\mathcal{IU}'})$$

$$(G^{r\,-1}\, B^{r\,-1}\, K^{crs\,-1}\, V^{obs}) = \underline{X^r}\, P\, V^{\underline{\mathcal{QUV}}} \quad \text{(+resolve amb)}$$

  - Revise G^r, using $\mathcal{IQU}$ (parallel-hands):

$$(B^{r\,-1}\, K^{crs\,-1}\, V^{obs}) = \underline{G^r}\, (P\, V^{\mathcal{IQU}})$$

  - Instrumental poln (D^r), using all of the above (cross-hands):

$$(G^{r\,-1}\, B^{r\,-1}\, K^{crs\,-1}\, V^{obs}) = \underline{D^r}\,(X^r\, P\, V^{\mathcal{IQU}})$$

  - (Iteration?)
- Correction:

$$V^{corr} = (P^{-1}\, X^{r\,-1}\, D^{r\,-1}\, G^{r\,-1}\, B^{r\,-1}\, K^{crs\,-1}\, V^{obs})$$

Parallel hands calibration

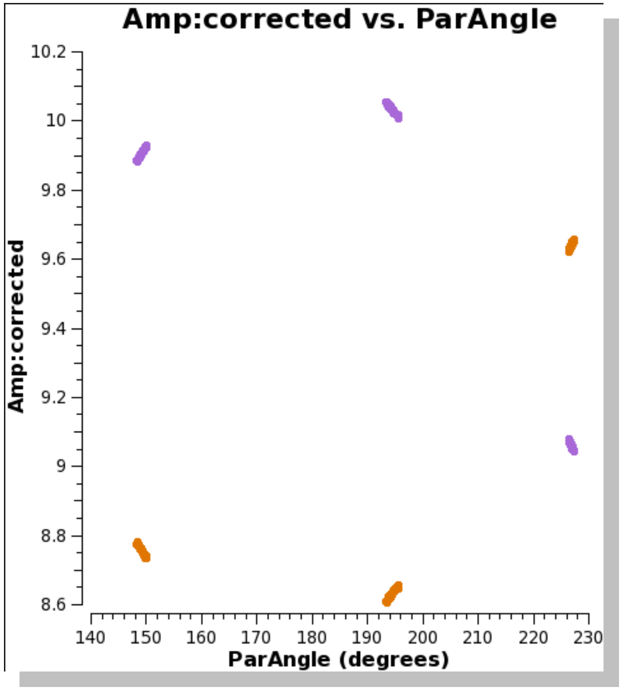
...as usual **setjy** and **bandpass**

gaincal phase and amplitude (calmode T = one solution for both pol) for all the calibrators

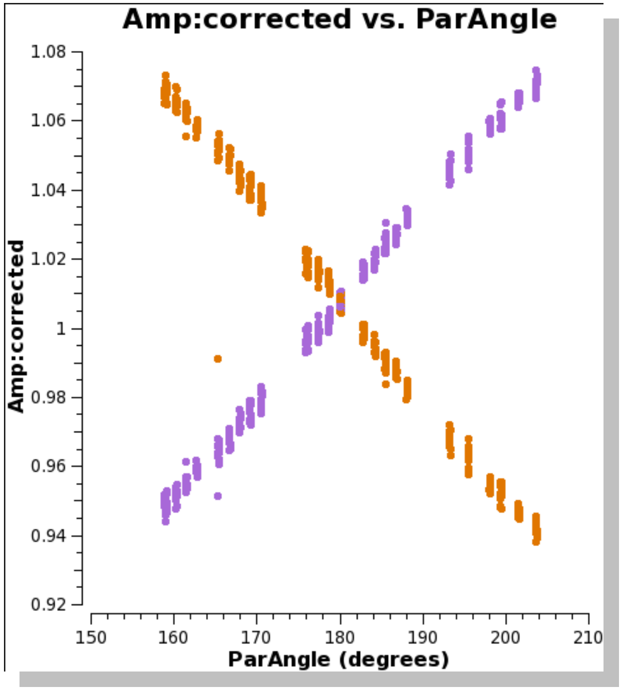
and **fluxscale** to bootstrap fluxes from that of Ceres

Results on the bandpass and phase calibrators:

3c279



J1310+3220



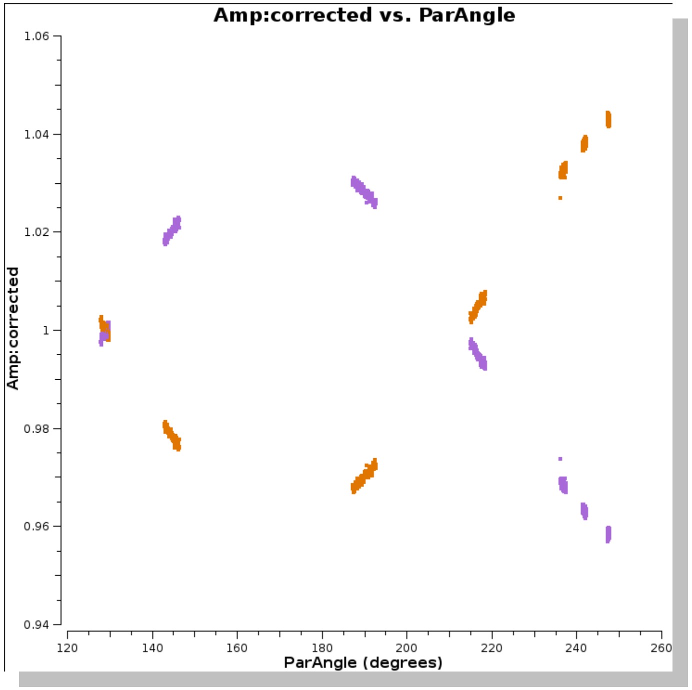
Parallel hands calibration

Using the same calibration on the pol calibrator

gaincal phase and amplitude (calmode T = one solution for both pol)

we would get:

J1337-1257



The polarization is clearly visible in the corrected data.

We determine gains for the polarization calibrator J1337-1257

gaincal (applying the bandpass on the fly) ==> gain table G1

Important parameters:

smodel = [1,0,0,0]
calmode = ap
gaintype = G

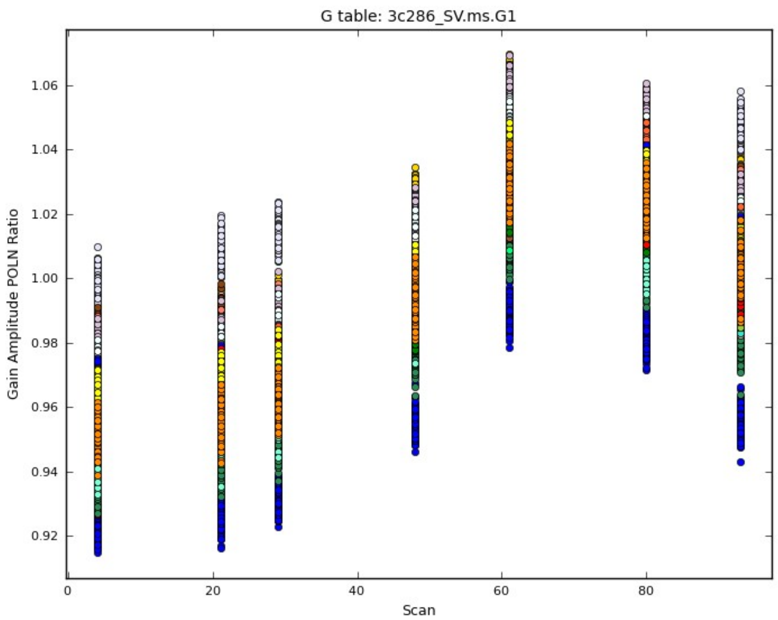
This are actually all default parameters in gaincal, but it is worth mentioning

- smodel assumes that the field is unpolarized, so the gains obtained will keep the polarization information

- the gains are solved contemporaneously for phase and amplitude

- gaintype = G the gains are determined for each polarization

Gaintable G1

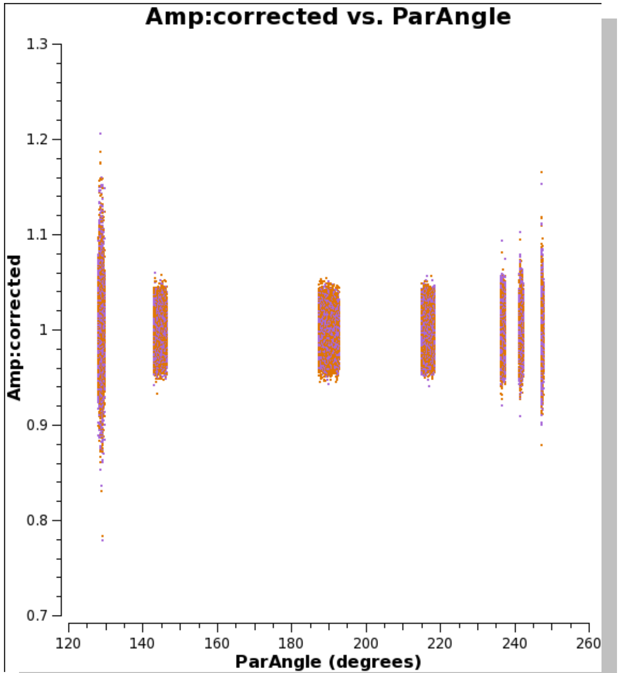


Because of
smodel =[1,0,0,0] & gaintype=G

the linear polarization-dependent solve makes gain “absorbe” Polarization. The gain ratio shows the sinusoidal variation (in parallactic angle)
clear sign of polarization in the gains

If we would apply this gaintable, since a solution for each polarization has been found

the difference between the two polarization would get corrected for.



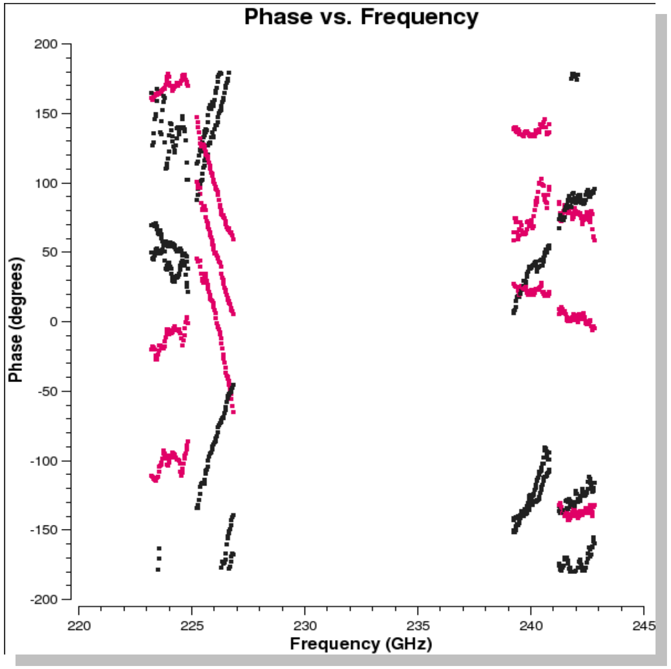
The gains contain the polarization information,
we extract it using the
qufromgain script included in the **almapolhelpers.py**

```
# In CASA
qu=qufromgain('3c286_SV.ms.G1')
```

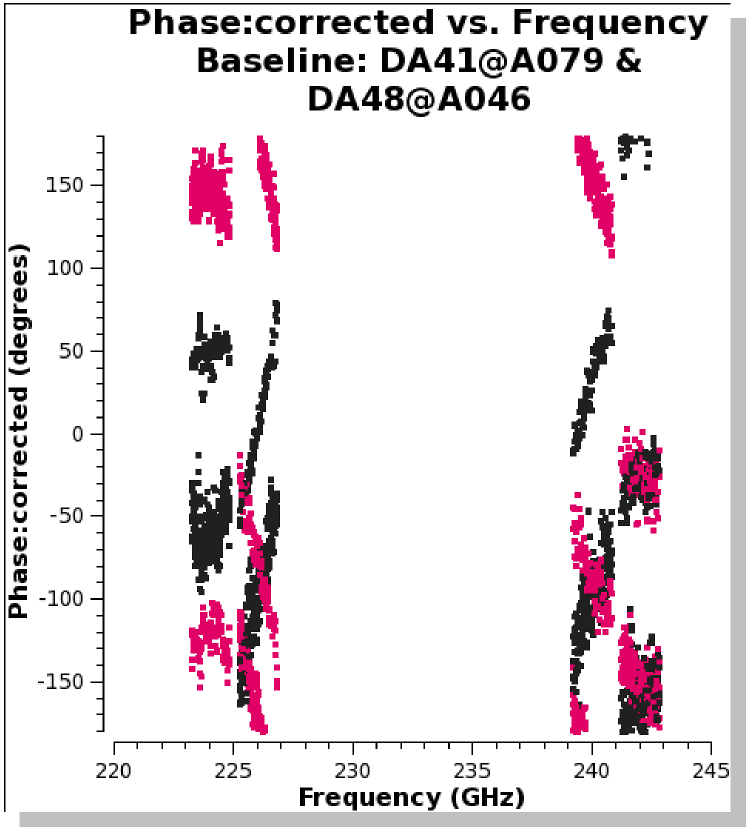
```
Latitude =  -23.0294371117
Found as many as 5 fields.
Found as many as 4 spws.
Fld= 0 Spw= 0 (B=06, PA offset=-135.0deg) Gx/Gy= 0.992405226704 Q= 0.0120290146946
               U= 0.0357751032462 P= 0.0377432802867 X= 35.7076525043
Fld= 0 Spw= 1 (B=06, PA offset=-135.0deg) Gx/Gy= 1.00460130437 Q= 0.0121161477985
               U= 0.0342484704653 P= 0.0363284842332 X= 35.2588247799
Fld= 0 Spw= 2 (B=06, PA offset=-135.0deg) Gx/Gy= 1.00477793757 Q= 0.0132551417327
               U= 0.0365817946839 P= 0.0389092082244 X= 35.0411959343
Fld= 0 Spw= 3 (B=06, PA offset=-135.0deg) Gx/Gy= 0.99729825908 Q= 0.013549612772
               U= 0.0372694902549 P= 0.0396561081062 X= 35.0104353733
For field id =  0  there are  4 good spws.
Spw mean: Fld= 0 Q= 0.0127374792495 U= 0.0359687146626 (rms= 0.000673704330994 0.00112523439036 )
          P= 0.038157460766 X= 35.2497952168
```

We get the J1337-1257 polarization in each spw out of the gains.
mean values : Q= 0.0127 U=0.0359 P=0.038 X=35.24

The cross-hand delay



The cross-hands
for the strongly polarized **3c279**
show a phase slope with
frequency



lower S/N for our polarization
calibrator **J1337-1257**
but slope still visible

The cross-hand delay

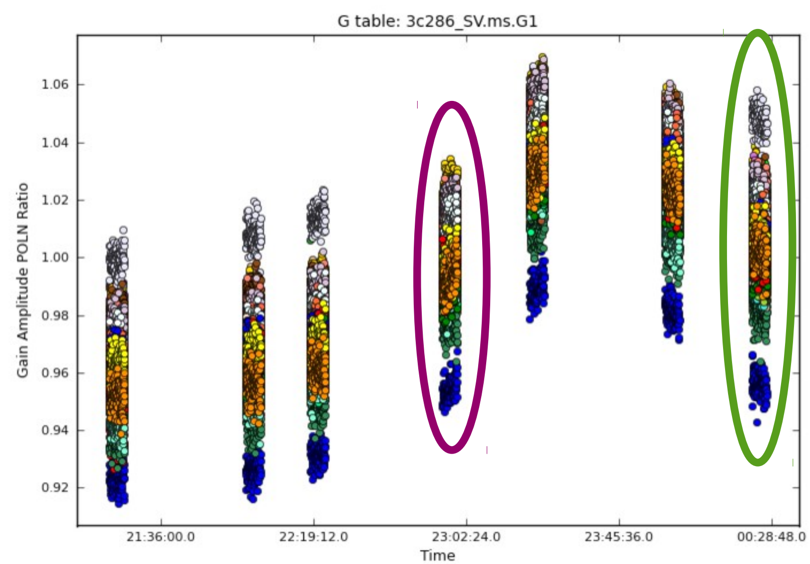
To solve for it we need to identify a scan where the source's cross-hand contribution is a maximum, since this will minimize the effect of instrumental polarization.

$$V_{XX} = \mathcal{I} + (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi)$$
$$V_{XY} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) + i\mathcal{V}$$
$$V_{YX} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) - i\mathcal{V}$$
$$V_{YY} = \mathcal{I} - (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi)$$

The cross-hand polarization terms are the derivative wrt ψ (and time) of the parallel hand.
So the cross-hand polarisation is maximum at the same time that the slope of the source pol contribution to the parallel hands is the largest.

The upward and downward slopes correspond to scan **48** and **93**

we use the **scan 48**



The cross-hand delay

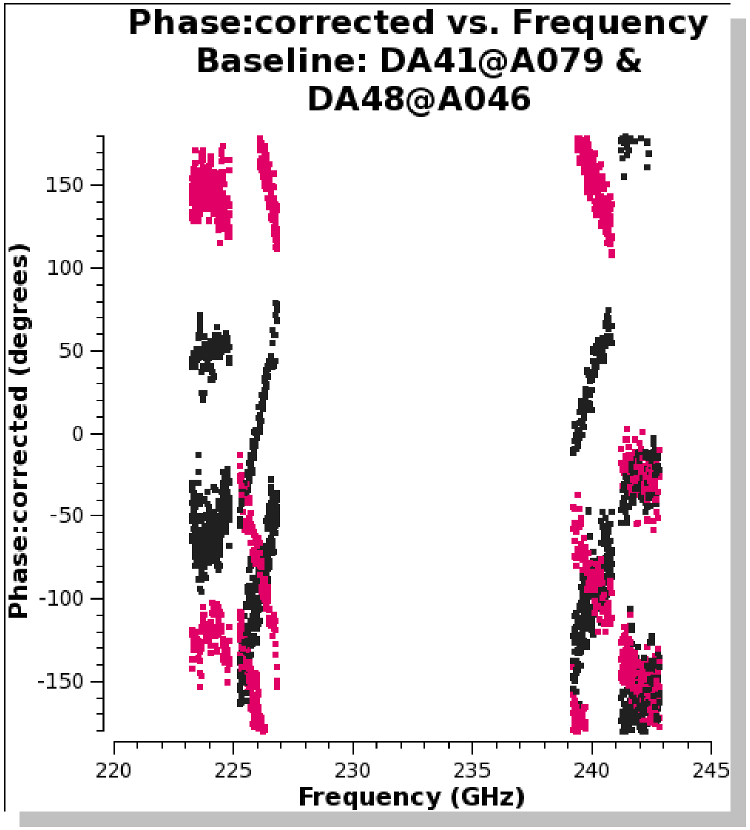
```
os.system('rm -rf 3c286_SV.ms.Kcrs')
gaincal(vis='3c286_SV.ms',
        caltable='3c286_SV.ms.Kcrs',
        selectdata=T,
        scan='48',
        gaintype='KCROSS',
        solint='inf',refant=refant,
        smodel=[1,0,1,0],
        gaintable=['3c286_SV.ms.Bscan','3c286_SV.ms.G1'],
        interp=['nearest','linear'])
```

smodel = [1,0,1,0]
gaintype='KCROSS'
scan='48'

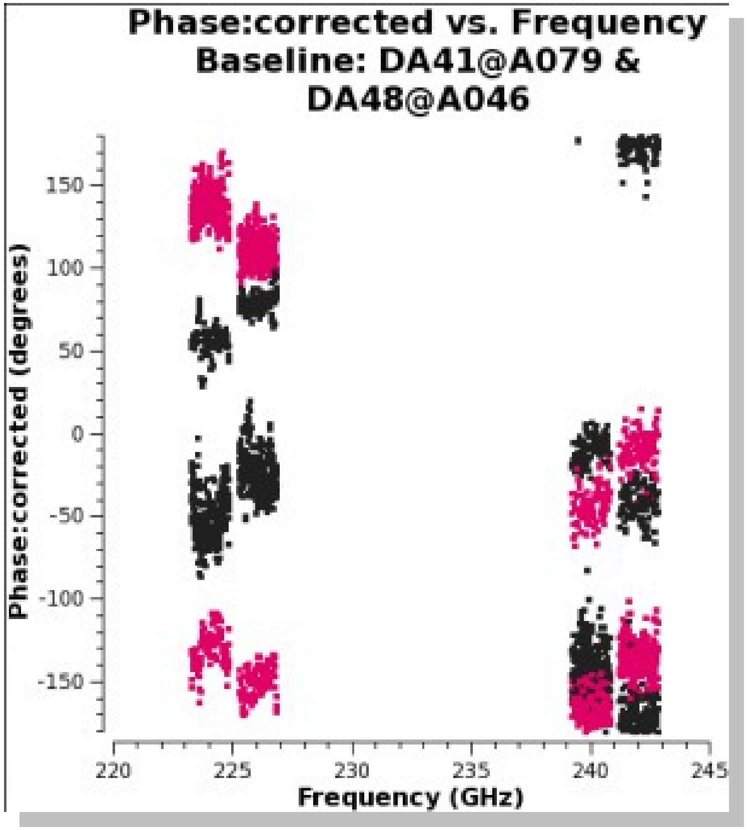
- The model this time assumes a non-zero source polarization signature in the cross-hands. So when calculating the gains in the ratio between data and model the model will be 1 for U.
- It is not relevant the value of the polarization stokes parameters since here we are just solving for the delay

The cross-hand delay correction

B, G1



B, G1, Kcrs



XY-phase and QU

An artifact of gain calibration refant

We don't measure absolute G and B
We fix to zero in both polarizations the phases of a reference antenna.
Differences among antennas in each polarization are preserved
---> no effect on parallel-hand calibration

But the refant's cross-hand bandpass phase remains undetected and uncorrected.

$$X^r = \begin{matrix} e^{i\rho} & 0 \\ 0 & 1 \end{matrix}$$

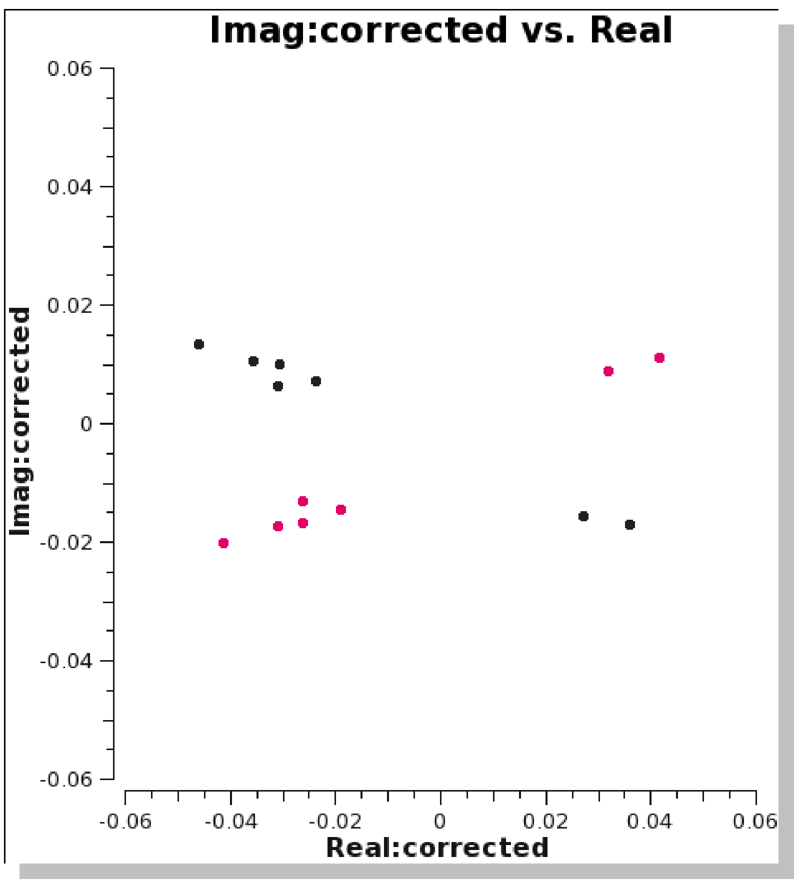
We need to correct for it in order to be able to combine the cross and parallel hands to extract the correct Stokes.

Solving for X^r and $\mathcal{Q},\mathcal{U}$

$$(G^{r\,-1}\,B^{r\,-1}\,V^{obs}) = X^r\,D\,P\,V^{mod}$$

- Consider just the gain- and bandpass-calibrated cross-hands:

$$V_{XY} = e^{i\rho}\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yj}{}^{*}+d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}{}^{*}-d_{Xi})\}$$
$$V_{YX} = e^{-i\rho}\,\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yi}+d_{Xj}{}^{*}) + \mathcal{Q}_\psi\,(d_{Yi}-d_{Xj}{}^{*})\}$$



Averaged all baselines and all channels (we applied Kcr so the channel average is coherent)

The XY and YX have the same slope with opposite signs because they are complex conjugate of each other.

Solving for $\boldsymbol{X^r}$ and $\mathcal{Q}, \mathcal{U}$ (cont)

- Average over baselines and correlations:

$$(\langle V_{XY}\rangle + \langle V_{YX}^*\rangle)/2 = \mathcal{U}_\psi e^{i\rho} + \langle \mathcal{I}(d_{Yj}^* + d_{Xi}) + \mathcal{Q}_\psi (d_{Yj}^* - d_{Xi})\rangle e^{i\rho}$$

$$= (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) e^{i\rho} + \varepsilon(t)$$

  - $\varepsilon(t)$ is small if d's are small and ~random
  - Requires non-zero $\mathcal{Q}, \mathcal{U}$
  - Measurements at (at least) 3 distinct ψ sufficient to determine ρ, $\mathcal{Q}$, $\mathcal{U}$, ε
    - gaincal(gaintype='XYf+QU')
  - Ambiguity: $(\rho, \mathcal{Q}, \mathcal{U}) \rightarrow (\rho+\pi, -\mathcal{Q}, -\mathcal{U})$
    - Resolvable using $\mathcal{Q}, \mathcal{U}$ estimate from gain ratio: `xyamb`
  - Requires $\boldsymbol{X^r}$ stability: a "good" refant for gain and bandpass (c.f. $|g_X/g_Y|$ stability expectations)

X-Y phase and QU

We need to estimate XY-phase offset and source polarization from the cross-hands

```
# In CASA
os.system('rm -rf 3c286_Band6.ms.XY0amb')
gaincal(vis='3c286_Band6.ms',caltable='3c286_Band6.ms.XY0amb',
          field='0',
          gaintype='XYf+QU',
          solint='inf',
          combine='scan,obs',
          preavg=300,
          refant=refant,
          smodel=[1,0,1,0],
          gaintable=['3c286_Band6.ms.Bscan','3c286_Band6.ms.G1','3c286_Band6.ms.Kcrs'],
          interp=['nearest','linear','nearest'])
```

smodel=[1,0,1,0]
gaintype='XYf+QU'
caltable='XY0amb'
preavg=300

- **gaintype='XYf+QU'** it averages all baselines together and first solves for a channelized XY-phase **(XYf)**, then corrects the slope and solves for a channel-averaged source polarization **(QU)**
- **Preavg=300** This limits averaging in the solution interval within the scan. The parallactic angle changes from scan to scan are important for the solution.
- The output table contains the values of Q,U and X-Y phase.
  A degeneracy may still be present in solving for the XY angle (X-Y phase, Q,U) --> (XY phase + π, Q, U)

X-Y phase and QU (resolving ambiguity)

To remove the ambiguity we use the **xyamb** module of the **almapolhelpers.py**

```
# In CASA
S=xyamb(xytab='3c286_Band6.ms.XY0amb',qu=qu[0],xyout='3c286_Band6.ms.XY0')
```

For each spw, it compares the Q and U signs obtained from the gaincal
to those previously extracted from the gains
and
converts the angle if necessary

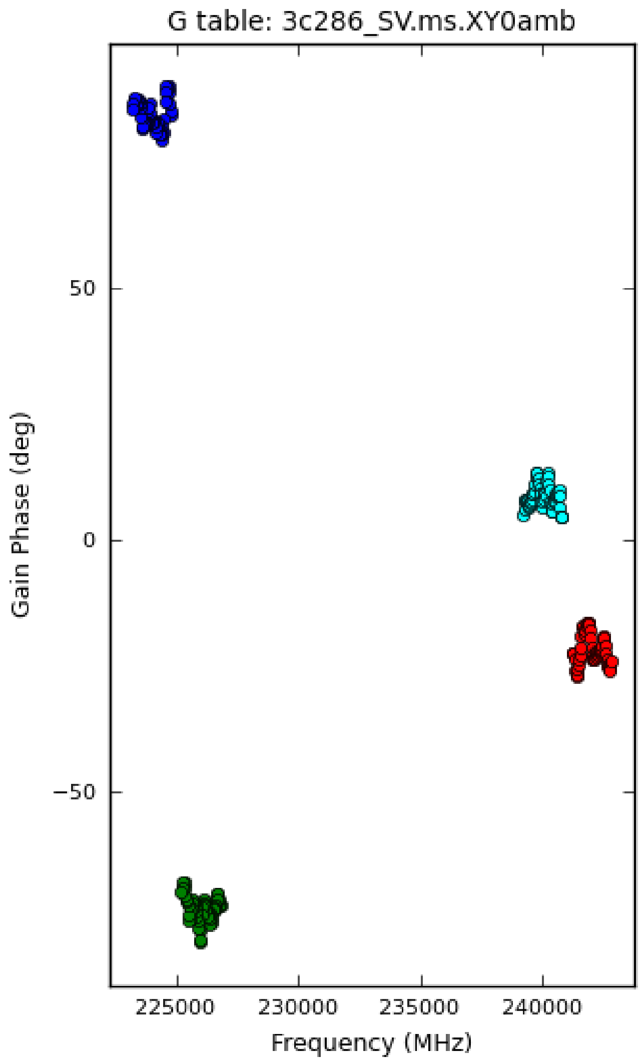
```
Expected QU =  (0.012737479249452035, 0.035968714662568159)
Spw = 0: Found QU = [ 0.01259385  0.04101928]
      ...KEEPING X-Y phase 84.4169495884 deg
Spw = 1: Found QU = [-0.01261223 -0.04112571]
   ...CONVERTING X-Y phase from -73.064852046 to 106.935147954 deg
Spw = 2: Found QU = [ 0.01355364  0.0418143 ]
      ...KEEPING X-Y phase 8.60411636622 deg
Spw = 3: Found QU = [ 0.01344682  0.04199271]
      ...KEEPING X-Y phase -21.7400889849 deg
Ambiguity resolved (spw mean): Q= 0.0130516346544 U= 0.0414879983291
 (rms= 0.000450227601812 0.000421946318322 ) P= 0.0434925185809 X= 36.2685199328
Returning the following Stokes vector: [1.0, 0.013051634654402733, 0.041487998329102993, 0.0]
```

the Stokes parameters obtained are stored in the S_july variable

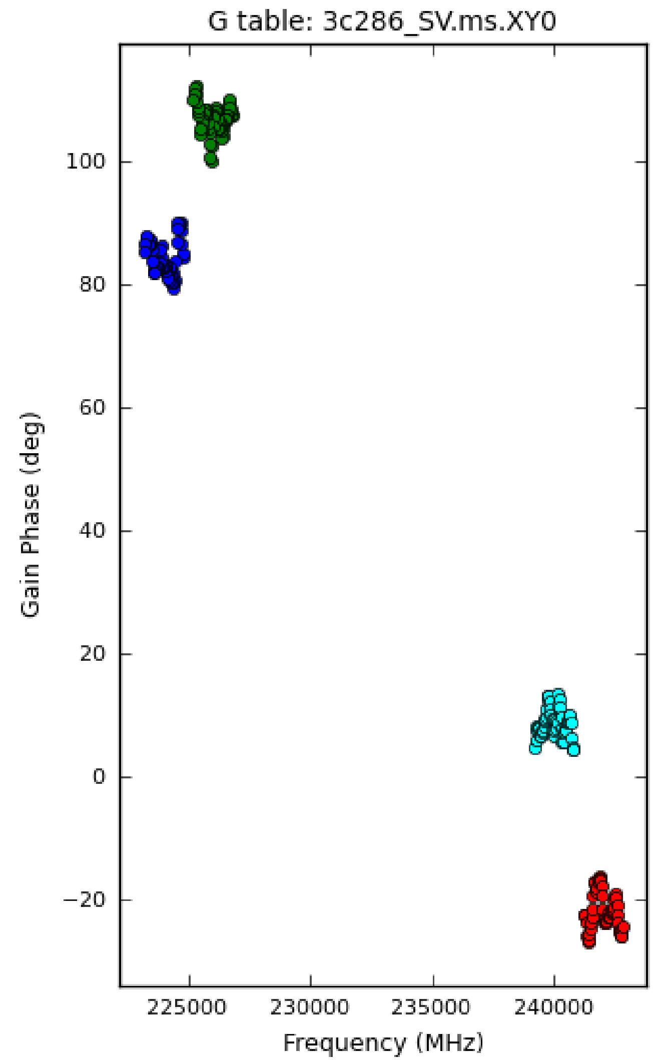
X-Y phase and QU (resolving ambiguity)

X-Y Phases in each spw

before



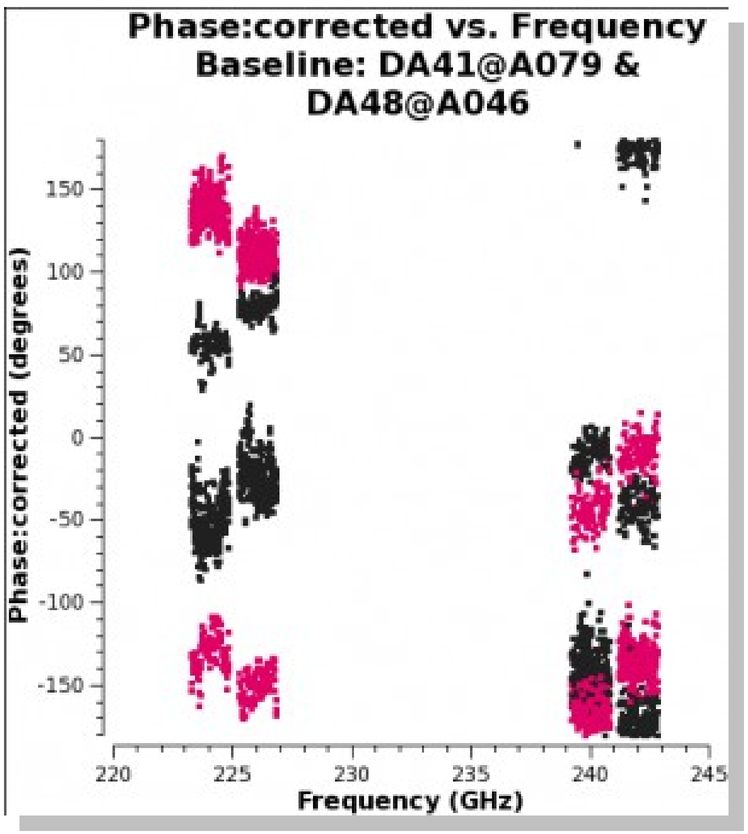
after



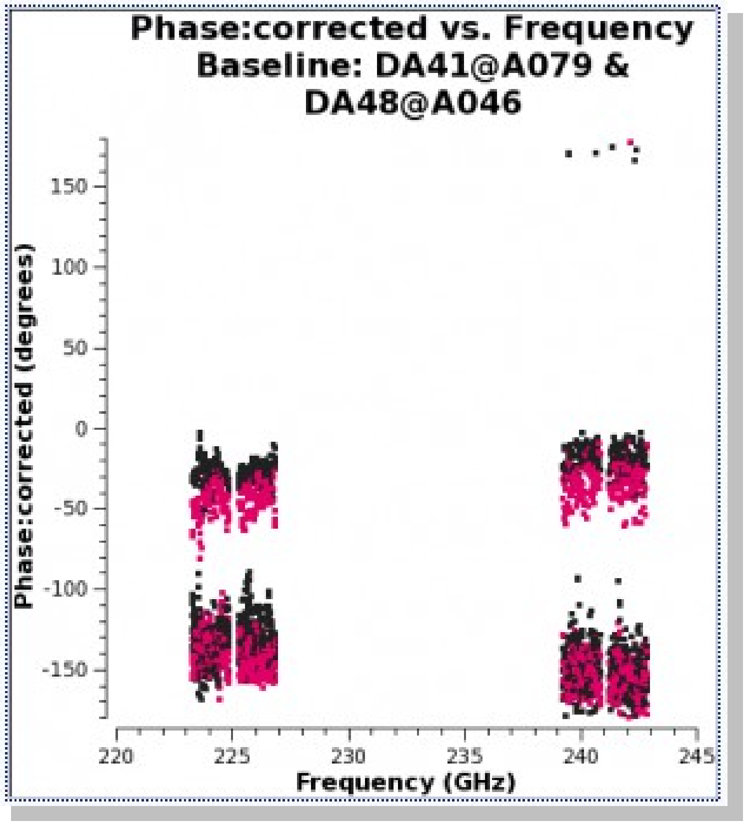

```
Spw = 1: Found QU = [-0.01262234 -0.04114043]
   ...CONVERTING X-Y phase from -72.99433926 to 107.00566074 deg
```

The X-Y phase and QU correction

B, G1,Kcrs

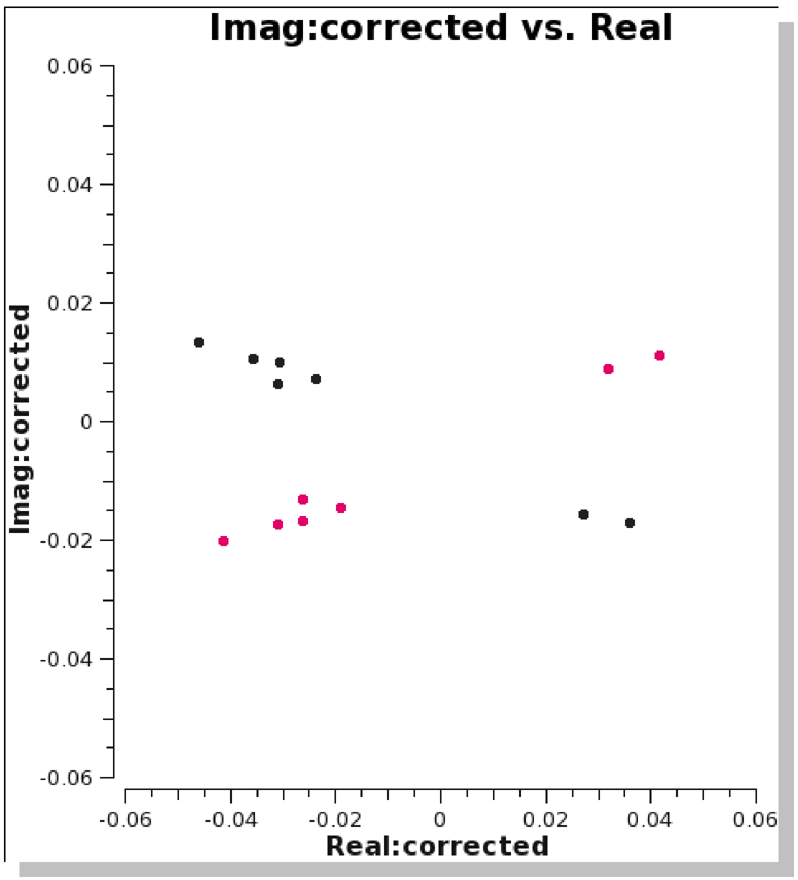


B, G1, Kcrs, XY0

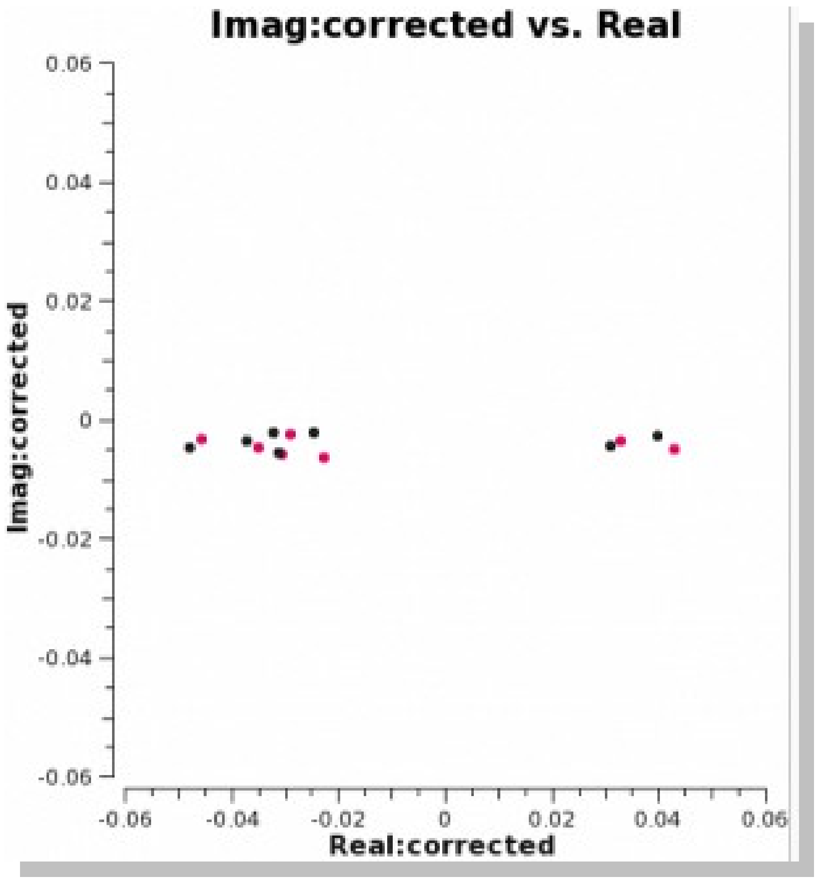


The X-Y phase and QU correction

B, G1,Kcrs



B, G1, Kcrs, XY0



Revise gains for the polarization calibrator

We now revise the gain calibration using the full polarization source model derived so far.

```
# In CASA
os.system('rm -rf 3c286_Band6.ms.G2.polcal')
gaincal(vis='3c286_Band6.ms',
        caltable='3c286_Band6.ms.G2.polcal',
        field='0',
        solint='int',
        refant=refant,
        smodel=S,
        gaintable=['3c286_Band6.ms.Bscan'],interp=['nearest'],
        parang=T)
```

smodel=S
gaintype=default=G

parang=T

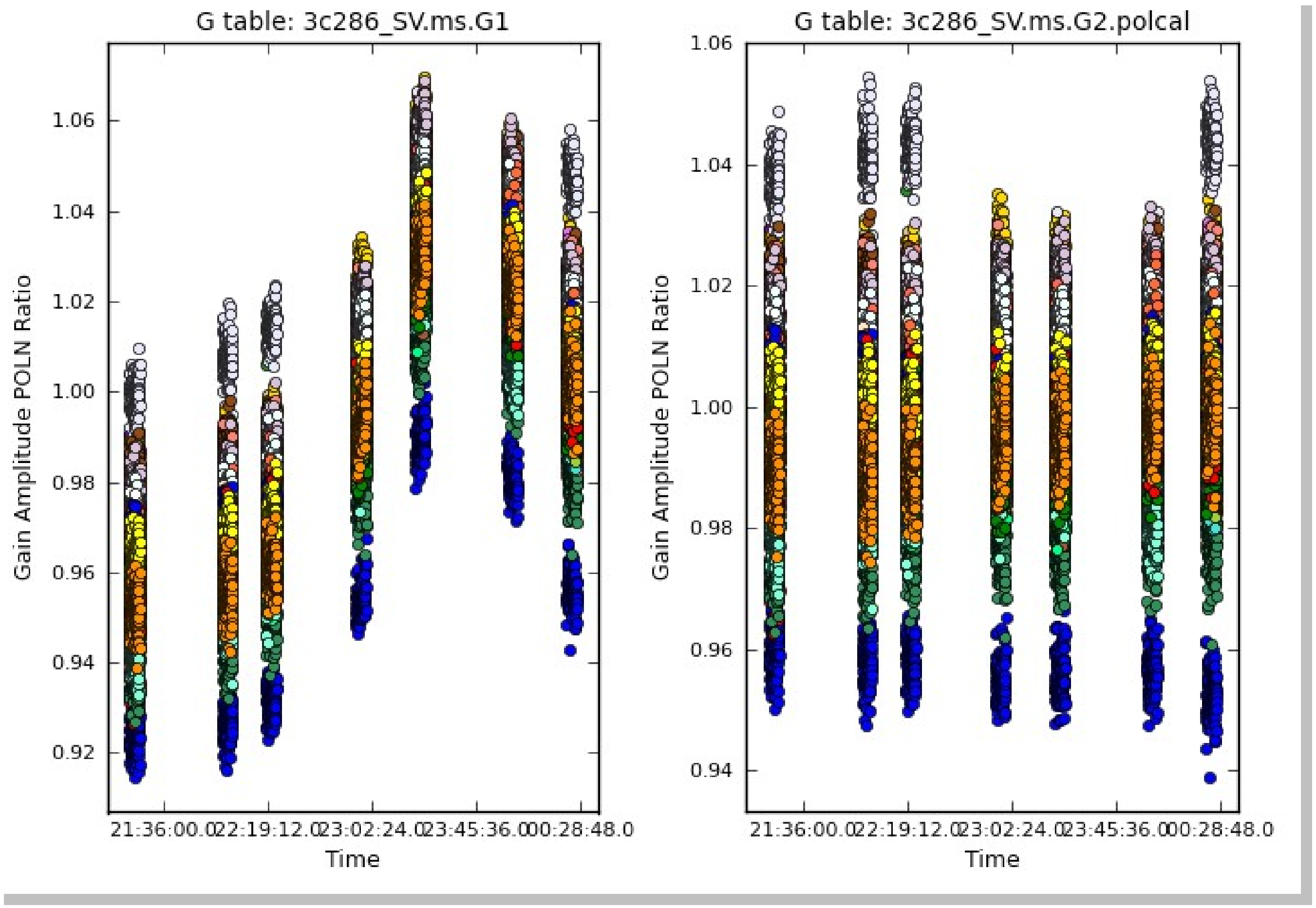
We actually don't use any of the cross-hand cal tables but

- **smodel=S**
  we use the model obtained before, including the actual polarization of the source
- **parang=T**
  in this way the supplied source linear polarization is properly rotated in the parallel-hand visibility model

Revise gains for the polarization calibrator

The gains now do not contain the polarization anymore.
If we use **qufromgain** on this table we get:

```
Spw mean: Fld= 0 Q= 0.000838681858308 U= -0.000234770753915 (rms= 0.000600891446491 0.000705059408831 )
          P= 0.000870921676357 X= -7.81921742693
```



And the sinusoidal distortion due to the polarization is gone!

Solving for the Leakage terms

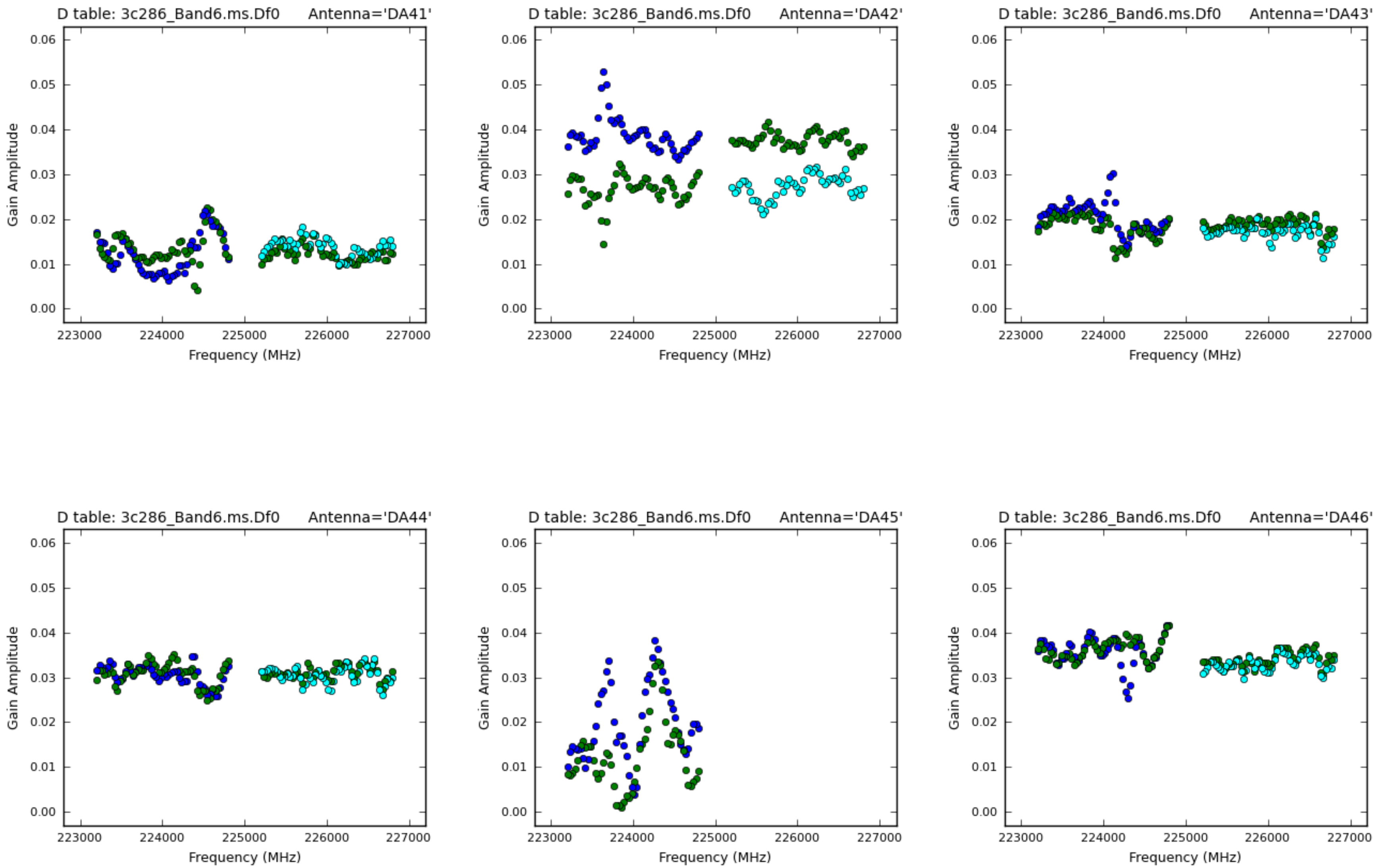
We use the task **polcal** to estimate the D-terms

```
# In CASA
os.system('rm -rf 3c286_Band6.ms.Df0*')
polcal(vis='3c286_Band6.ms',
       caltable='3c286_Band6.ms.Df0',
       field='0', #J1337-1257
       solint='inf',combine='obs,scan',
       preavg=300,
       poltype='Dflls',
       refant='', #solve absolute D-term
       smodel=S,
       gaintable=['3c286_Band6.ms.Bscan','3c286_Band6.ms.G2.polcal','3c286_Band6.ms.Kcrs', '3c286_Band6.ms.XY0'],
       gainfield=['', '', '', ''],
       interp=['nearest','linear','nearest','nearest'])
```

smodel=S
poltype=Dflls
refant=''
preavg=300

- **smodel=S** again, the model to be used in the gain calculation is the one we got before, including polarization
- **poltype=Dflls** frequency dependent solver
- **Refant=' '** No reference antenna is really important here. Assuming a refant would mean assume its X feed perfect, so by referring all instrumental polarization to it, we would in fact discard valid information about the imperfections in the refant feed.

Solving for the Leakage terms



Before applying the leakage terms
Need to relabeling to enforce full matrix correction

```
# In CASA
Dgen(dtab='3c286_Band6.ms.Df0',dout='3c286_Band6.ms.Df0gen')
```

Applying all: B, G2polcal, Kcrs, XY0, Df0gen

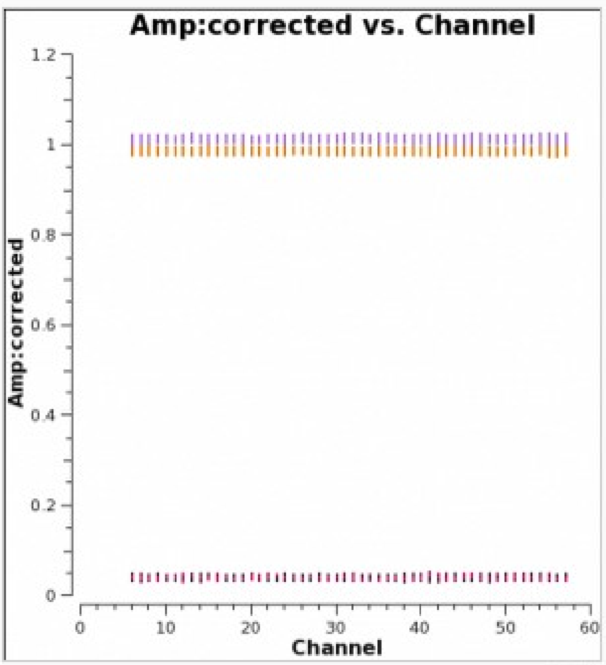


Figure 24. XX,YY, XY and YX amp vs channels for all antennas after the whole calibration.

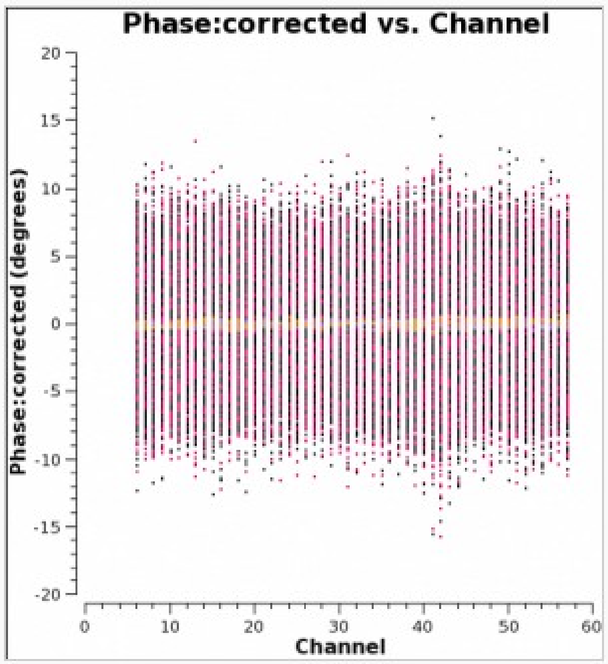


Figure 25. XX,YY, XY and YX amp vs channels for all antennas after the whole calibration

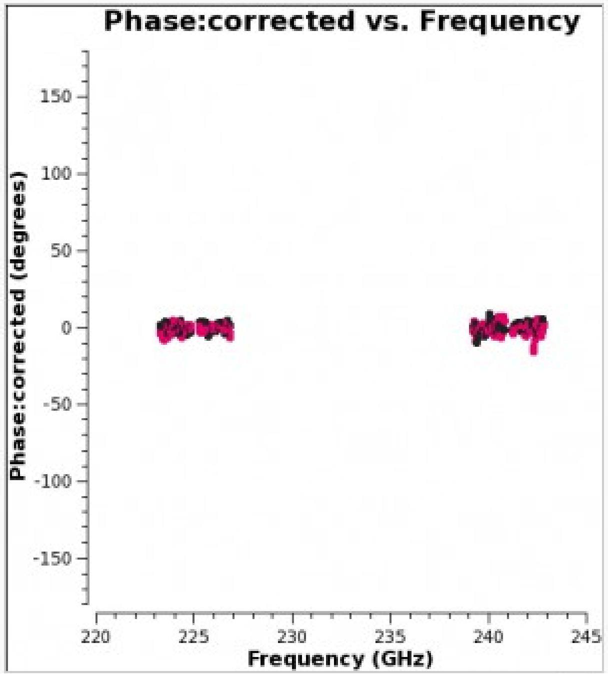


Figure 26. Baseline DA41&DA48: XY and YX phase vs frequency after the whole calibration. All scans are averaged.