Synthesis Polarimetry Fundamentals for ALMA

George Moellenbrock (NRAO)

2013 June 25

ALMA Italian ARC Node

References

- Synthesis Imaging in Radio Astronomy II (1998 NRAO Summer School Book)
- Thompson, Moran, & Swenson
- Sault, Killeen, & Kesteven 1991, "AT Polarization Calibration" (memo)
- Hamaker, Bregman, & Sault 1995

Synthesis Fundamentals

- Van-Cittert-Zernike Theorem:

$$\mathcal{F}t\,\{\mathcal{I}\} = V^{true} = \langle s_i\, s_j{}^{*}\rangle$$

- s_i is the direction-dependent EM field disturbance generated by an astronomical source as ideally sampled *at* antenna *i*
- It is actually a vector quantity, $\boldsymbol{s}_i$
- And it must be calibrated:

$$\boldsymbol{V}^{obs} = \langle \boldsymbol{x}_i\, \boldsymbol{x}_j{}^{*}\rangle$$

$$\boldsymbol{x}_i = \boldsymbol{J}_i \boldsymbol{s}_i$$

Polarization Bases

- To sample the incident *vector* EM field with optimum accuracy and precision, we arrange to receive dual (nominally) orthogonal polarizations at each antenna
- *Any* orthogonal basis will suffice; we typically choose one of two standard bases:
  - Linear: X, Y (e.g., ALMA)
  - Circular: R, L (e.g., EVLA > 1GHz)
- Basis choice influences observation patterns and calibration heuristics, in practice

Correlations and Stokes Parameters - I

- Measured *correlations* or *visibilities* are 4-vectors:

$$
\begin{aligned}
V_{XX} &= \mathcal{I} + \mathcal{Q} & \quad V_{RR} &= \mathcal{I} + \mathcal{V} \\
V_{XY} &= \mathcal{U} + i\mathcal{V} & \quad V_{RL} &= \mathcal{Q} + i\mathcal{U} \\
V_{YX} &= \mathcal{U} - i\mathcal{V} & \quad V_{LR} &= \mathcal{Q} - i\mathcal{U} \\
V_{YY} &= \mathcal{I} - \mathcal{Q} & \quad V_{LL} &= \mathcal{I} - \mathcal{V}
\end{aligned}
$$

- Total Intensity (Stokes $\mathcal{I}$) always in parallel-hands
- Linear Polarization (Stokes $\mathcal{Q},\mathcal{U}$):
  - linear basis: parallel- and cross-hands
  - circular basis: cross-hands
- Circular Polarization (Stokes $\mathcal{V}$):
  - linear basis: cross-hands
  - circular basis: parallel hands

Correlations and Stokes Parameters - II

- *Calibrated* correlation visibilities are combined to form the Stokes visibilities:

$$\mathcal{I} = (V_{XX} + V_{YY})/2 \quad = (V_{RR} + V_{LL})/2$$
$$\mathcal{Q} = (V_{XX} - V_{YY})/2 \quad = (V_{RL} + V_{LR})/2$$
$$\mathcal{U} = (V_{XY} + V_{YX})/2 \quad = (V_{RL} - V_{LR})/2i$$
$$\mathcal{V} = (V_{XY} - V_{YX})/2i = (V_{RR} - V_{LL})/2$$

  – All four correlations must be *consistently* calibrated, so the combinations can be formed correctly

- Stokes *visibilities* may then be inverted to form Stokes *images*

Generic Matrix Calibration

- Scalar: $x = J\, s$
- Vector: $$\begin{pmatrix} x_p \\ x_p \end{pmatrix} = \begin{pmatrix} J_{p \leftarrow p} & J_{p \leftarrow q} \\ J_{q \leftarrow p} & J_{q \leftarrow q} \end{pmatrix} \begin{pmatrix} s_p \\ s_p \end{pmatrix}$$
  - Polarization may mix!
  - J_{pq}, J_{qp} generally small, by design, but not strictly zero

Factored Calibration

$$\boldsymbol{J_i\, s_i = B_i\, G_i\, D_i\, P_i\, T_i\, s_i}$$

$\boldsymbol{B_i}$: bandpass

$\boldsymbol{G_i}$: (electronic) gain $\boldsymbol{D_i}$: instrumental polarization

$\boldsymbol{T_i}$: troposphere $\boldsymbol{P_i}$: parallactic angle

- Approximately "physical"
  - OK to imagine each antenna's received signal, $\boldsymbol{s_i}$, corrupted by terms in order from right to left, though actual effects are not strictly quite so discrete
- Really a convenient factorization of effects that organizes the matrix algebra
  - Consistent with hardware design (e.g., polarizer in front of amplifiers, etc.)
  - Order is important!
- Notation in this talk:
  - One subscript: antenna-based
  - Two subscripts: baseline-based
  - Zero subscripts: mnemonic or 'operator' notation
  - Lower case: vector or matrix element

The Measurement Equation

$$\boldsymbol{V^{obs} = B\ G\ D\ P\ T\ V^{true}}$$

$$\boldsymbol{V^{corr} = T^{-1}\ P^{-1}\ D^{-1}\ G^{-1}\ B^{-1}\ V^{obs}}$$

- Where:

  $\boldsymbol{V^{obs}}$ = observed visibility 4-vector (correlations)

  $\boldsymbol{V^{true}}$ = true visibility 4-vector; estimated by V^{mod} when solving

  $\boldsymbol{V^{corr}}$ = corrected visibility 4-vector

  B, G, D, P, T = baseline-based operators each representing pair-wise combinations of antenna-based terms
- We will solve for relevant calibration terms with an appropriate heuristic
- Each solve has the form:

$$\boldsymbol{(V^{obs}) = \underline{J}\ (V^{mod})}$$

  – ***()*** indicates partial correction/corruption by already-known terms

Gain-like terms: T, G, B

- Troposphere: $\boldsymbol{T} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix}$
  - Scalar
  - Commutes with everything; often absorbed in ***G***
  - WVR phase is a variety of ***T***
- "Electronic" gain: $\boldsymbol{G} = \begin{pmatrix} g_X & 0 \\ 0 & g_Y \end{pmatrix}$
  - Diagonal
  - The ubiquitous term: typically describes many instrumental effects
- Bandpass: $\boldsymbol{B} = \begin{pmatrix} b_X(\nu) & 0 \\ 0 & b_Y(\nu) \end{pmatrix}$
  - Diagonal:
  - Frequency-dependent version of ***G***
  - ALMA Tsys is a variety of ***B***

Calibration is a Bootstrapping Process

- Assume unity for all "non-relevant" terms
- Adopt priors, if available, e.g., B^{tsys}, T^{wvr}
- Solve for the dominant term, e.g., ***G:***

$$(B^{tsys\ -1}\ V^{obs}) = \underline{G}\ (T^{wvr}\ Vmod)$$

- Then, using ***G***, solve for next-most-important term, ***B***:

$$(B^{tsys\ -1}\ V^{obs}) = \underline{B}\ (G\ T^{wvr}\ V^{mod})$$

  - ***B*** will be a function of how good net prior calibration is
- Iterate and extend, as necessary (generalized 'selfcal')
  - E.g., use ***B*** to obtain ***G*** for other sources, timescales
  - Revise source models, as needed (traditional 'selfcal')
- Correct the observed data (~sufficient for total intensity):

$$V^{corr} = (T^{wvr\ -1}\ G^{-1}\ B^{-1}\ B^{tsys\ -1}\ V^{obs})$$

Scalar Treatment and Poln Basis

- Linear basis: If calibrator has non-zero (and unknown) linear polarization, polarization-dependent gain-like solves will absorb it, e.g.:

$$g_X' = g_X(1 + \mathcal{Q}/\mathcal{I})^{0.5} \quad g_Y' = g_Y(1 - \mathcal{Q}/\mathcal{I})^{0.5}$$

  - Parallel hands will be corrected for *calibrator* polarization!
  - Cross-hand correction error is 2nd order in $\mathcal{Q}$: $(1 - \mathcal{Q}^2/\mathcal{I}^2)^{0.5}$
    - Distortion is small (we will rely on this later)
  - Formally, desirable to measure *G* on an unpolarized source, depend on $|g_X/g_Y|$ stability, and use (unpol) $T(t)$
  - $\mathcal{Q}$ is systematically time-dependent (due to parallactic angle), so the apparent gain ratio provides a means of estimating source polarization if sufficient sampling available, and true gain ratio is stable:

$$g_X'/g_Y' = (g_X/g_Y)\,(1 + \mathcal{Q}/\mathcal{I})^{0.5}/(1 - \mathcal{Q}/\mathcal{I})^{0.5}$$
$$\approx (g_X/g_Y)\,(1 - 2\mathcal{Q}/\mathcal{I})^{0.5}$$

- Circular basis: $\mathcal{Q} \rightarrow \mathcal{V}$; Usually, $\mathcal{V} = 0$, so no problem

Parallactic Angle, *P*

- Sky orientation rotates in the field of view of an alt-az telescope:

$$\psi(t) = \frac{\cos b \sin H(t)}{\sin b \cos \delta - \cos b \sin \delta \cos H(t)}$$

b = latitude; H(t) = Hour Angle; δ = declination

$$\boldsymbol{P}^{lin} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \qquad \boldsymbol{P}^{circ} = \begin{pmatrix} e^{-i\psi} & 0 \\ 0 & e^{i\psi} \end{pmatrix}$$

- At $\psi = 0$ (meridian: $H = 0$), mechanical feed position angle may be offset (linear basis)

$\boldsymbol{P}$ in the Linear Basis

- If $\psi_i = \psi$ for all antennas (small array: ~uniform *b* and *H*), the trigonometry of $\boldsymbol{P}$ greatly simplified
  - Uniform rotation of linear polarization
  - Cross-hand rotation ($\mathcal{U}_\psi$) in quadrature w/ parallel-hand rotation ($\mathcal{Q}_\psi$):

$$
\begin{aligned}
V_{XX} &= \mathcal{I} + (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) &&= \mathcal{I} + \mathcal{Q}_\psi \\
V_{XY} &= (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) + i\mathcal{V} &&= \mathcal{U}_\psi + i\mathcal{V} \\
V_{YX} &= (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) - i\mathcal{V} &&= \mathcal{U}_\psi - i\mathcal{V} \\
V_{YY} &= \mathcal{I} - (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) &&= \mathcal{I} - \mathcal{Q}_\psi
\end{aligned}
$$

$\boldsymbol{P}$ in the Circular Basis

- If $\psi_i = \psi$ for all antennas (small array), uniform rotation of linear polarization occurs only in the cross-hands:

$$V_{RR} = \mathcal{I} + \mathcal{V}$$

$$V_{RL} = (\mathcal{Q} + i\mathcal{U})\, e^{-i2\psi}$$

$$V_{LR} = (\mathcal{Q} - i\mathcal{U})\, e^{+i2\psi}$$

$$V_{LL} = \mathcal{I} - \mathcal{V}$$

Instrumental Polarization, *D*

- Each polarized receptor sees some of the other polarization:

$$\boldsymbol{D} = \begin{pmatrix} 1 & d_X(\nu) \\ d_Y(\nu) & 1 \end{pmatrix}$$

  - General
  - Notation: "d_X" is "the fraction of Y polarization sensed by X"
  - N.B.: On-diagonal effects factored into ***G, B***
- Origins:
  - Finite impurities in polarizers
  - Reflections that return in opposite polarization: standing waves
  - Asymmetry in optics induce spurious polarization (e.g., squint)

D: Properties

- Orthogonality condition: $(d_{Xi}+d_{Yi}^*) = 0$
  - Easier to achieve than purity
- Orthogonality estimator: $(d_{Xi}+d_{Yi}^*)/2$
- Purity estimator: $(d_{Xi}-d_{Yi}^*)/2$
- In the linear basis ($d << 1.0$):
  - *Real(d)* = linear polarization orientation error
  - *Imag(d)* = ellipticity error

$\boldsymbol{D}$ in the Linear Basis - I

$\boldsymbol{V = D\ P\ V^{true}}$:

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{U}_\psi+i\mathcal{V})d_{Xj}^* + d_{Xi}(\mathcal{U}_\psi-i\mathcal{V}) + d_{Xi}(\mathcal{I}-\mathcal{Q}_\psi)d_{Xj}^*$$

$$V_{XY} = (\mathcal{I}+\mathcal{Q}_\psi)d_{Yj}^* + (\mathcal{U}_\psi+i\mathcal{V}) + d_{Xi}(\mathcal{U}_\psi-i\mathcal{V})d_{Yj}^* + d_{Xi}(\mathcal{I}-\mathcal{Q}_\psi)$$

$$V_{YX} = d_{Yi}(\mathcal{I}+\mathcal{Q}_\psi) + d_{Yi}(\mathcal{U}_\psi+i\mathcal{V})d_{Xj}^* + (\mathcal{U}_\psi-i\mathcal{V}) + (\mathcal{I}-\mathcal{Q}_\psi)d_{Xj}^*$$

$$V_{YY} = d_{Yi}(\mathcal{I}+\mathcal{Q}_\psi)d_{Yj}^* + d_{Yi}(\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{U}_\psi-i\mathcal{V})d_{Yj}^* + (\mathcal{I}-\mathcal{Q}_\psi)$$

- Nice symmetries:
  - *d* multiplies pure cross-/parallel-hands in the parallel-/cross-hands
  - d^2 multiplies other pure parallel-/cross-hand (c.f. antenna-based description)

D in the Linear Basis - II

- Linearized, sorted:

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{U}_\psi+i\mathcal{V})d_{Xj}^* + d_{Xi}(\mathcal{U}_\psi-i\mathcal{V})$$

$$V_{XY} = (\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{I}+\mathcal{Q}_\psi)d_{Yj}^* + d_{Xi}(\mathcal{I}-\mathcal{Q}_\psi)$$

$$V_{YX} = (\mathcal{U}_\psi-i\mathcal{V}) + d_{Yi}(\mathcal{I}+\mathcal{Q}_\psi) + (\mathcal{I}-\mathcal{Q}_\psi)d_{Xj}^*$$

$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + d_{Yi}(\mathcal{U}_\psi+i\mathcal{V}) + (\mathcal{U}_\psi-i\mathcal{V})d_{Yj}^*$$

$\boldsymbol{D}$ in the Linear Basis -III

- Linearized, sorted, $d\mathcal{V} \sim 0$, regrouped Stokes

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Xj}^* + d_{Xi})$$

$$V_{XY} = (\mathcal{U}_\psi + i\mathcal{V}) + \mathcal{I}\,(d_{Yj}^* + d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}^* - d_{Xi})$$

$$V_{YX} = (\mathcal{U}_\psi - i\mathcal{V}) + \mathcal{I}\,(d_{Yi} + d_{Xj}^*) + \mathcal{Q}_\psi\,(d_{Yi} - d_{Xj}^*)$$

$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Yi} + d_{Yj}^*)$$

  - Properties:
    - Constant (per baseline) complex offset proportional to $\mathcal{I}$ in cross-hands
    - *d*-scaled time-dependent source linear polarization in all correlations

D in the Circular Basis - I

V = D P V^true^:

$$V_{RR} = (\mathcal{I}+\mathcal{V}) + (\mathcal{Q}+i\mathcal{U})e^{-i2\psi}d_{Rj}^* + d_{Ri}(\mathcal{Q}-i\mathcal{U})e^{+i2\psi} + d_{Ri}(\mathcal{I}-\mathcal{V})d_{Rj}^*$$

$$V_{RL} = (\mathcal{I}+\mathcal{V})d_{Lj}^* + (\mathcal{Q}+i\mathcal{U})e^{-i2\psi} + d_{Ri}(\mathcal{Q}-i\mathcal{U})e^{+i2\psi}d_{Lj}^* + d_{Ri}(\mathcal{I}-\mathcal{V})$$

$$V_{LR} = d_{Li}(\mathcal{I}+\mathcal{V}) + d_{Li}(\mathcal{Q}+i\mathcal{U})e^{-i2\psi}d_{Rj}^* + (\mathcal{Q}-i\mathcal{U})e^{+i2\psi} + (\mathcal{I}-\mathcal{V})d_{Rj}^*$$

$$V_{LL} = d_{Li}(\mathcal{I}+\mathcal{V})d_{Lj}^* + d_{Li}(\mathcal{Q}+i\mathcal{U})e^{-i2\psi} + (\mathcal{Q}-i\mathcal{U})e^{+i2\psi}d_{Lj}^* + (\mathcal{I}-\mathcal{V})$$

- Symmetries (same as linear basis):
  - *d* multiplies pure cross-/parallel-hands in parallel-/cross-hands
  - d^2 multiplies other pure parallel-/cross-hand

D in the Circular Basis - II

- Linearized, sorted:

$V_{RR} = (\mathcal{I}+\mathcal{V}) + (\mathcal{Q}+i\mathcal{U})e^{-i2\psi}d_{Rj}{}^* + d_{Ri}(\mathcal{Q}-i\mathcal{U})e^{+i2\psi}$

$V_{RL} = (\mathcal{Q}+i\mathcal{U})e^{-i2\psi} + (\mathcal{I}+\mathcal{V})d_{Lj}{}^* + d_{Ri}(\mathcal{I}-\mathcal{V})$

$V_{LR} = (\mathcal{Q}-i\mathcal{U})e^{+i2\psi} + d_{Li}(\mathcal{I}+\mathcal{V}) + (\mathcal{I}-\mathcal{V})d_{Rj}{}^*$

$V_{LL} = (\mathcal{I}-\mathcal{V}) + d_{Li}(\mathcal{Q}+i\mathcal{U})e^{-i2\psi} + (\mathcal{Q}-i\mathcal{U})e^{+i2\psi}d_{Lj}{}^*$

D in the Circular Basis - III

- Linearized, sorted, $d\mathcal{V} \sim 0$:

$$V_{RR} = (\mathcal{I}+\mathcal{V}) + (\mathcal{Q}+i\mathcal{U})e^{-i2\psi}d_{Rj}{}^* + d_{Ri}(\mathcal{Q}-i\mathcal{U})e^{+i2\psi}$$

$$V_{RL} = (\mathcal{Q}+i\mathcal{U})e^{-i2\psi} + \mathcal{I}(d_{Lj}{}^*+d_{Ri})$$

$$V_{LR} = (\mathcal{Q}-i\mathcal{U})e^{+i2\psi} + \mathcal{I}(d_{Li}+d_{Rj}{}^*)$$

$$V_{LL} = (\mathcal{I}-\mathcal{V}) + d_{Li}(\mathcal{Q}+i\mathcal{U})e^{-i2\psi} + (\mathcal{Q}-i\mathcal{U})e^{+i2\psi}d_{Lj}{}^*$$

- Traditionally, VLA also has ignored Q,U terms in V_{RR}, V_{LL}
  - Degenerate D solution: $(d_{Lj}-a)^*+(d_{Ri}+a^*) = (d_{Lj}{}^*+d_{Ri})$, so one d_R arbitrarily set to zero (refant)
  - Time-dependent closure errors in V_{RR}, V_{LL} for polarized sources: limits $\mathcal{I}$ dynamic range to 10-100K…

Visualizing Polarization Effects in the Linear Feed Basis

- Illustrates corruption by ***P, D, G*** in a single channel
- A 4.5h 'observation' of 3C286
- An idealized point source model (10% linear polarization):

  $\mathcal{I} = 1.0 \quad \mathcal{Q} = 0.06 \quad \mathcal{U} = 0.08 \quad \mathcal{V} = 0.0$

- No noise
- Baselines to a single antenna

Ideal Visibilities: $\boldsymbol{V^{true}}$

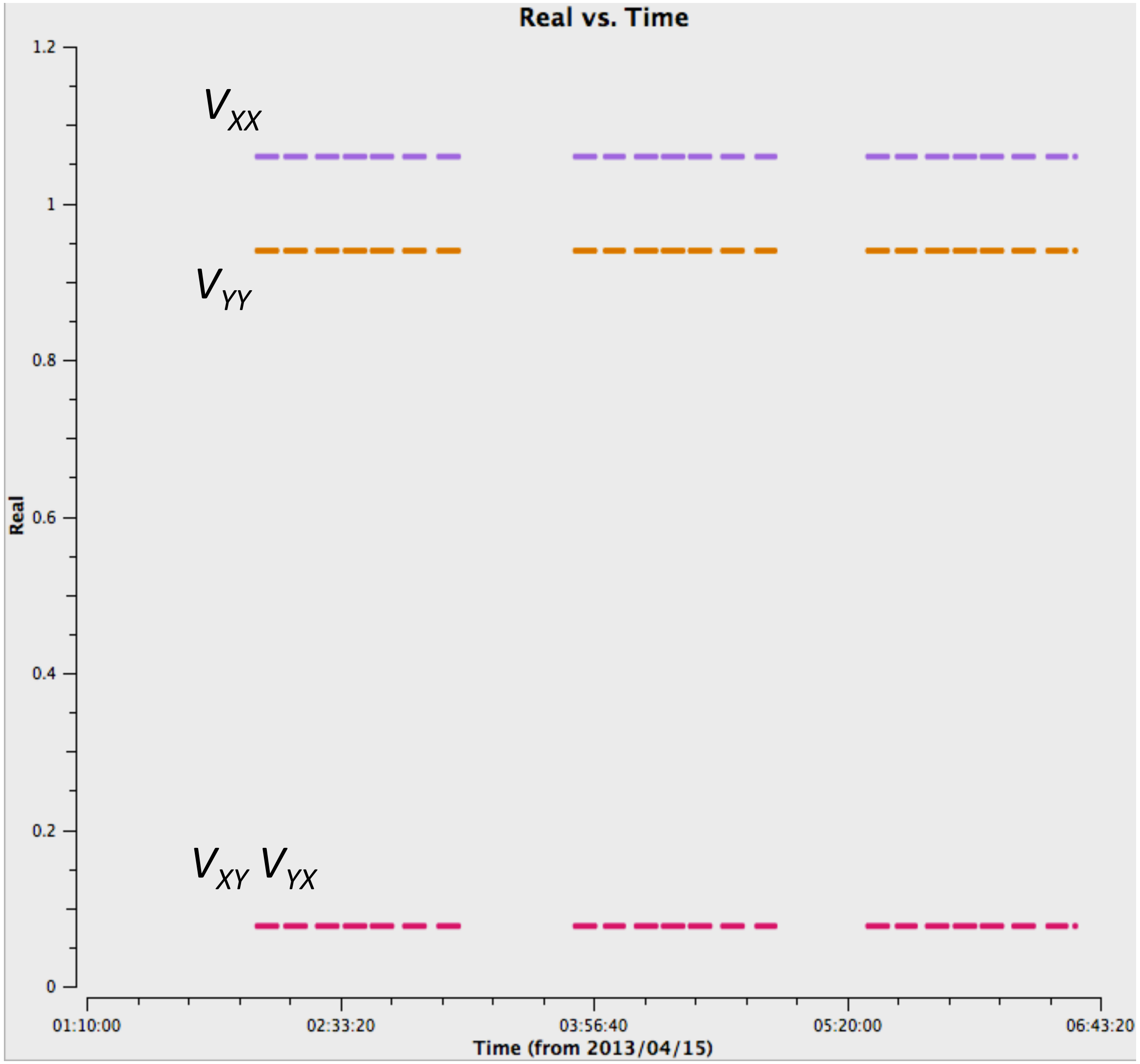
$$V_{XX} = \mathcal{I} + \mathcal{Q}$$

$$V_{XY} = \mathcal{U} + i\mathcal{V}$$

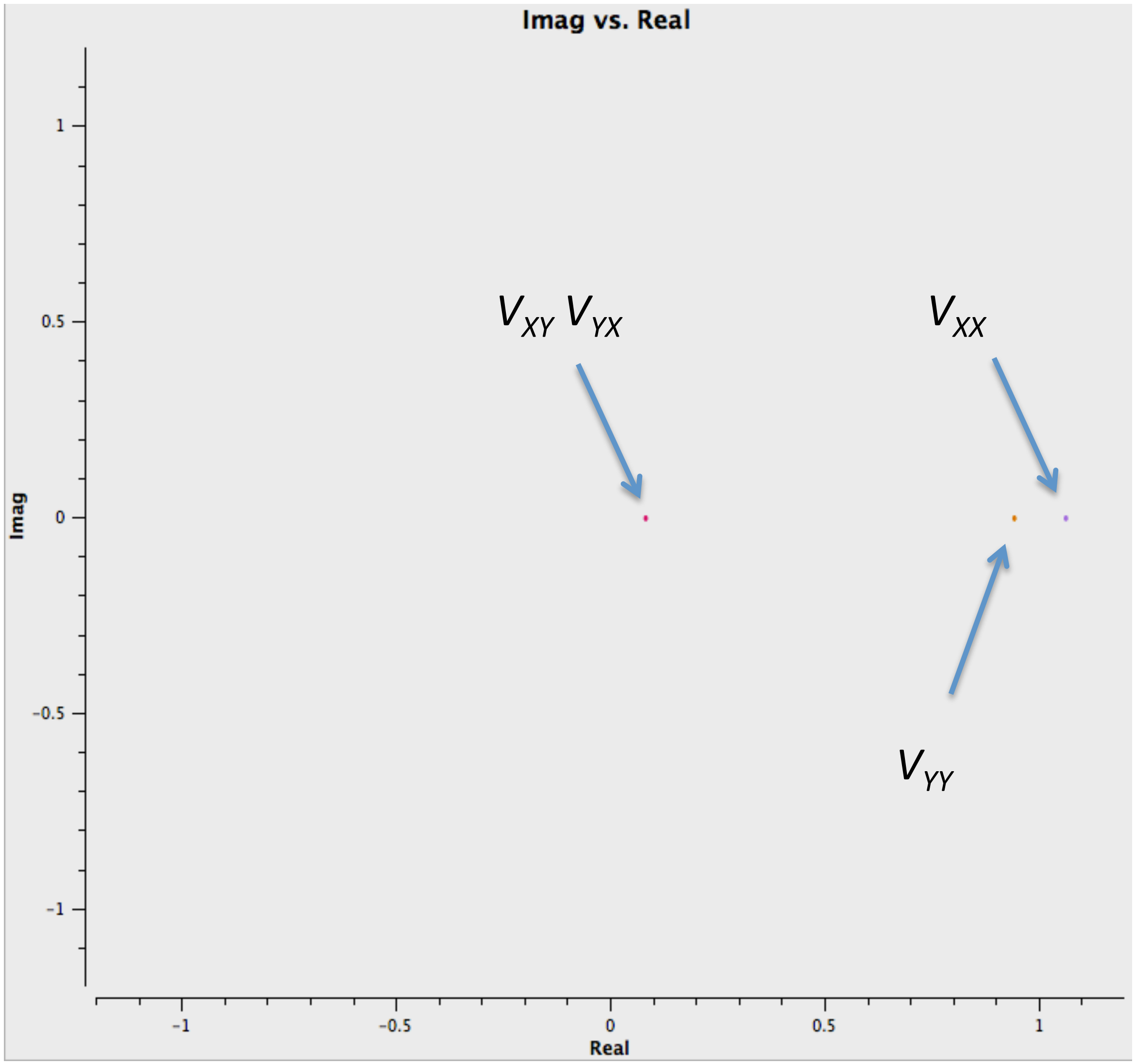
$$V_{YX} = \mathcal{U} - i\mathcal{V}$$

$$V_{YY} = \mathcal{I} - \mathcal{Q}$$

V^{true}: Real vs time



V^{true}: Imag vs. Real



Parallactic Angle: $\boldsymbol{P\,V^{true}}$

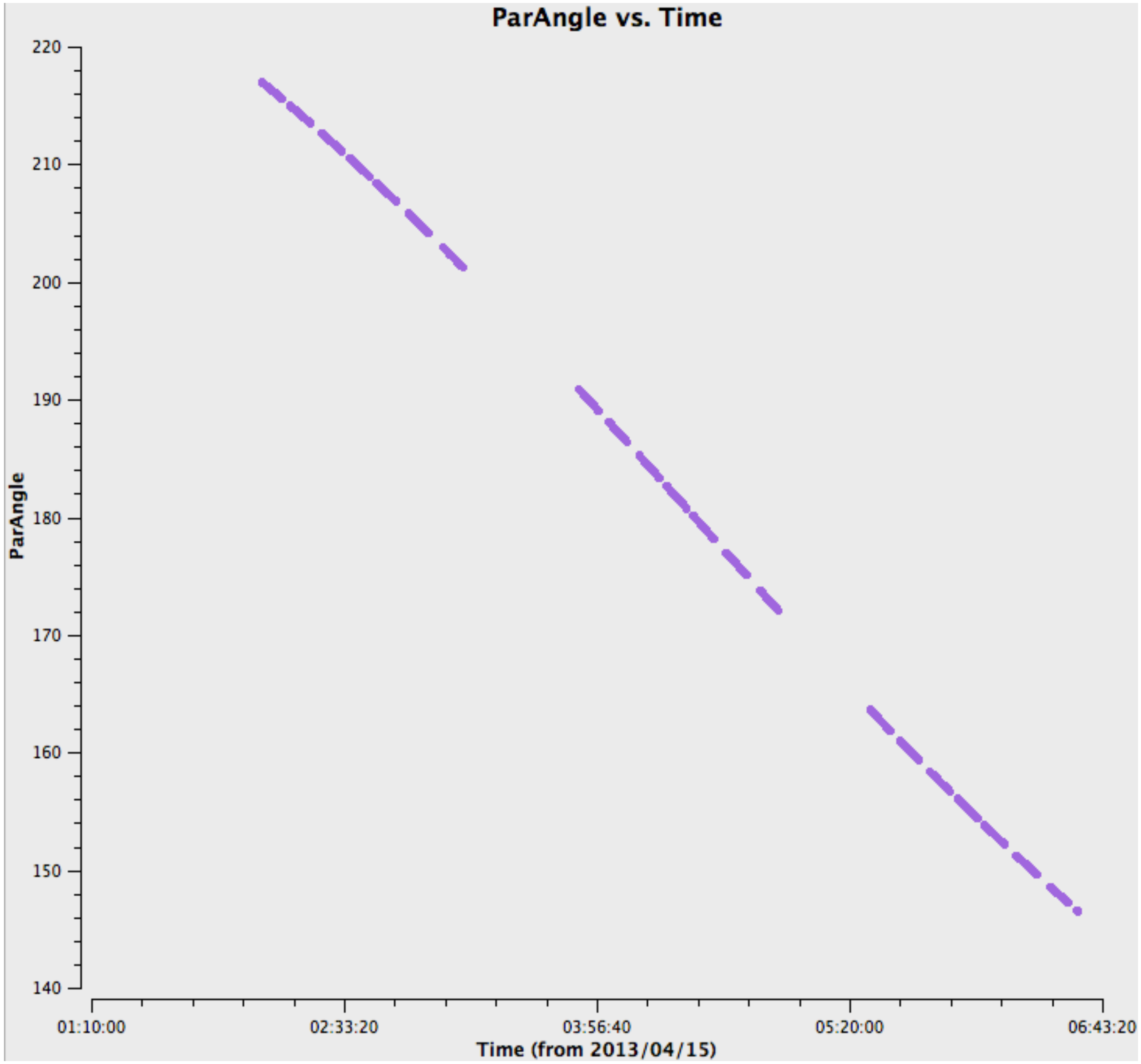
$$V_{XX} = \mathcal{I} + (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} + \mathcal{Q}_\psi$$

$$V_{XY} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) + i\mathcal{V} = \mathcal{U}_\psi + i\mathcal{V}$$

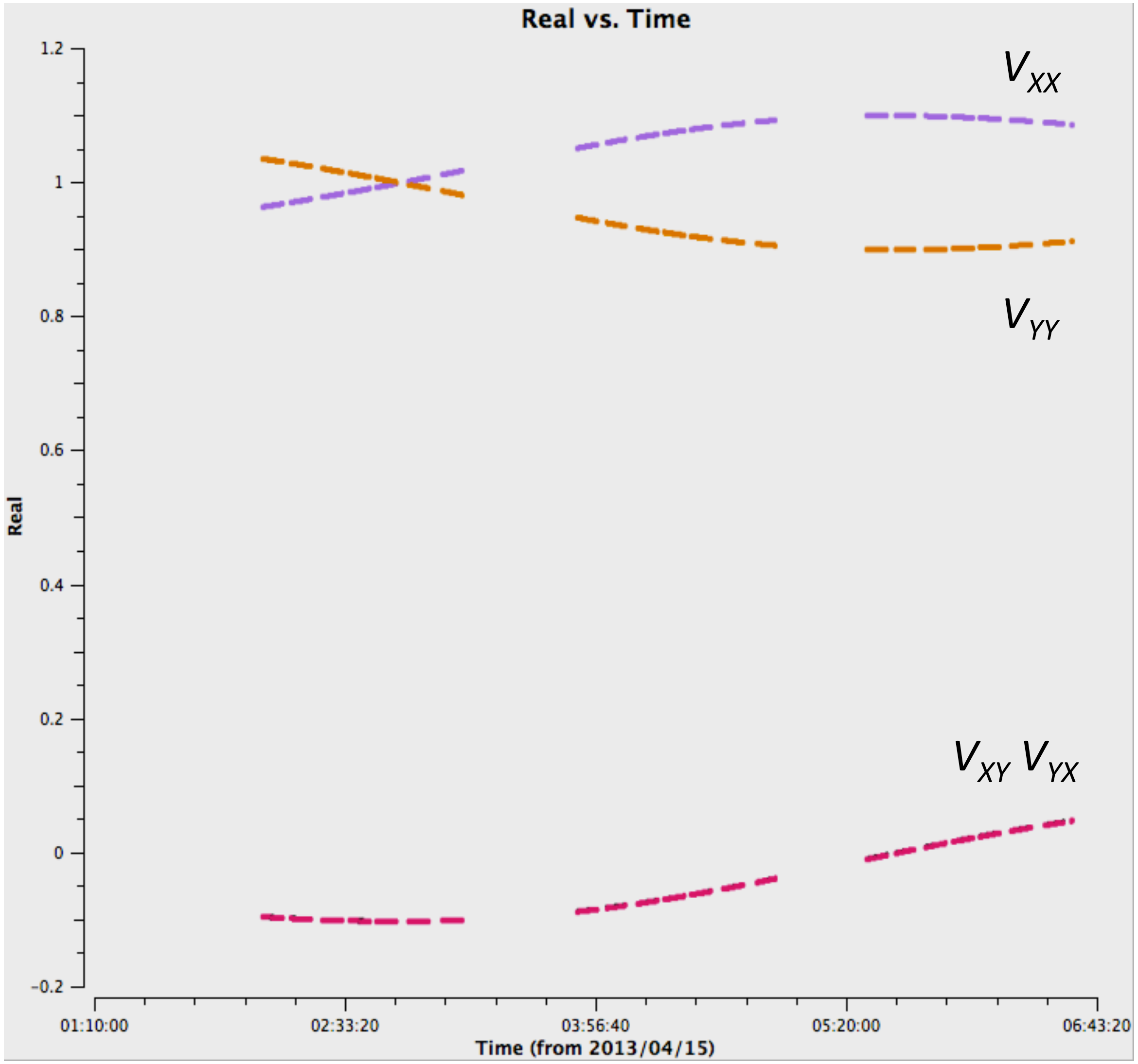
$$V_{YX} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) - i\mathcal{V} = \mathcal{U}_\psi - i\mathcal{V}$$

$$V_{YY} = \mathcal{I} - (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} - \mathcal{Q}_\psi$$

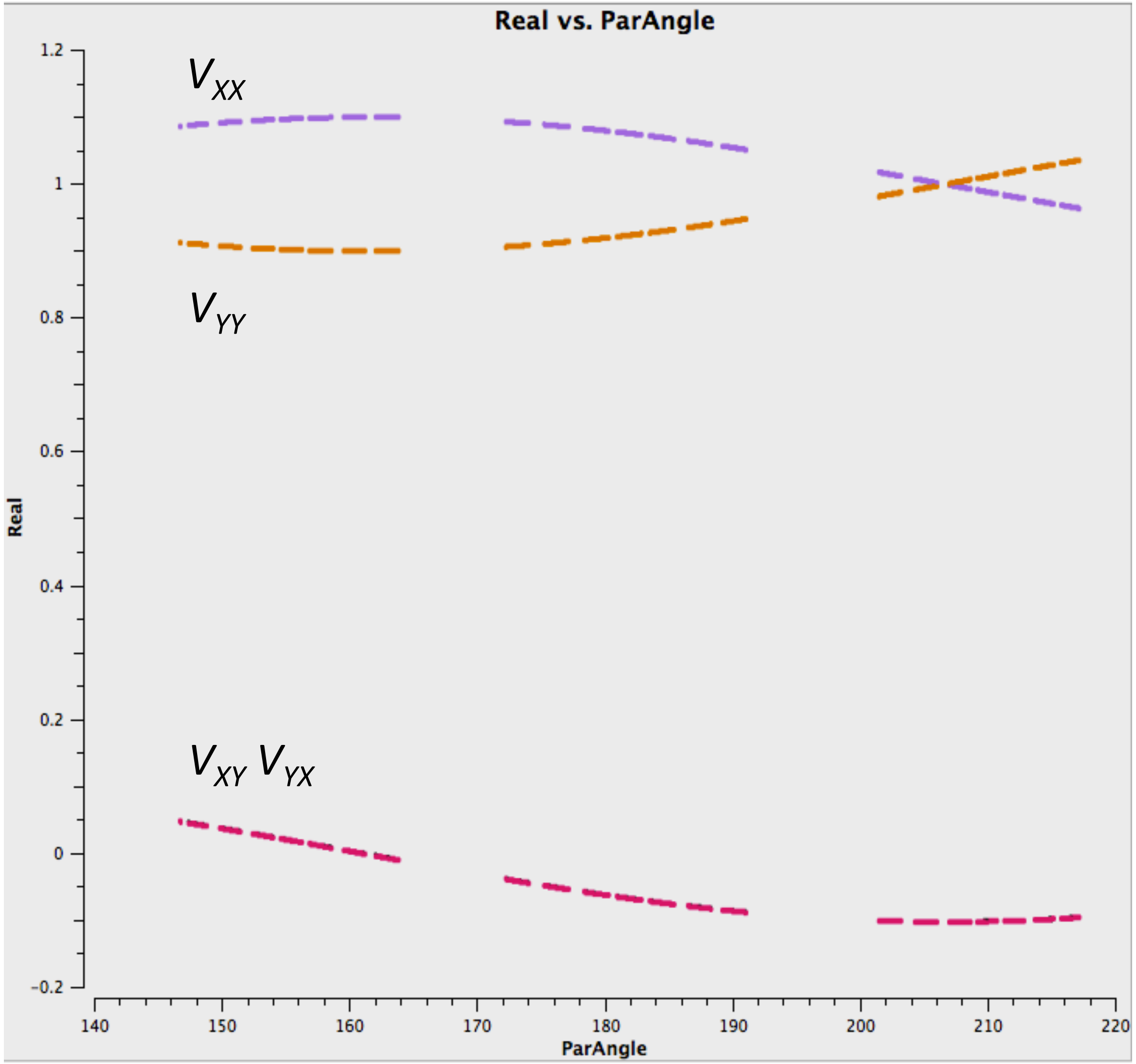
P: parang vs. time



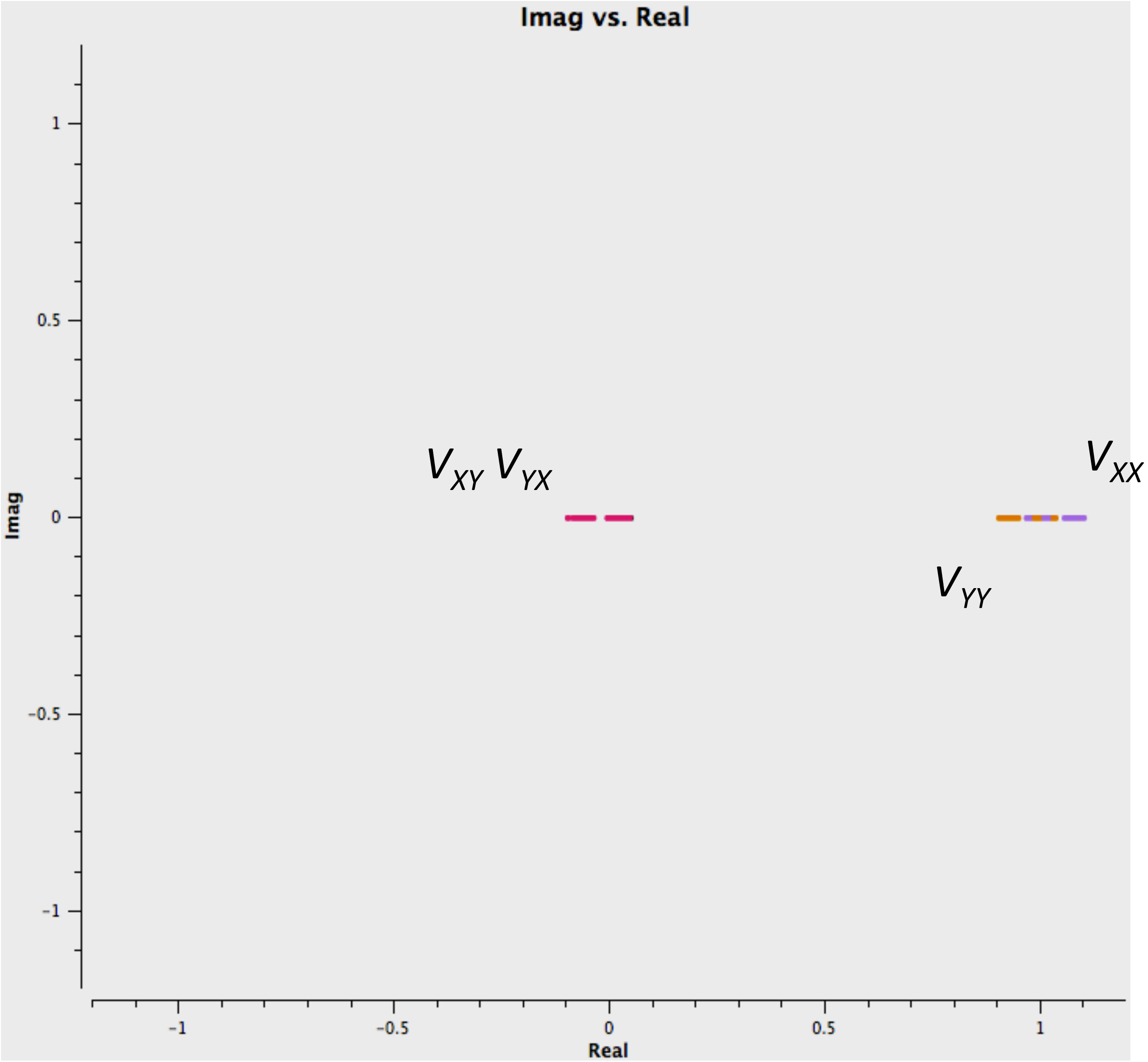
$P\ V^{true}$: Real vs. time



$P\ V^{true}$: Real vs. parang



$P\,V^{true}$: Imag vs. Real



Instrumental Polarization: $\boldsymbol{D\,P\,V^{true}}$

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Xj}{}^* + d_{Xi})$$

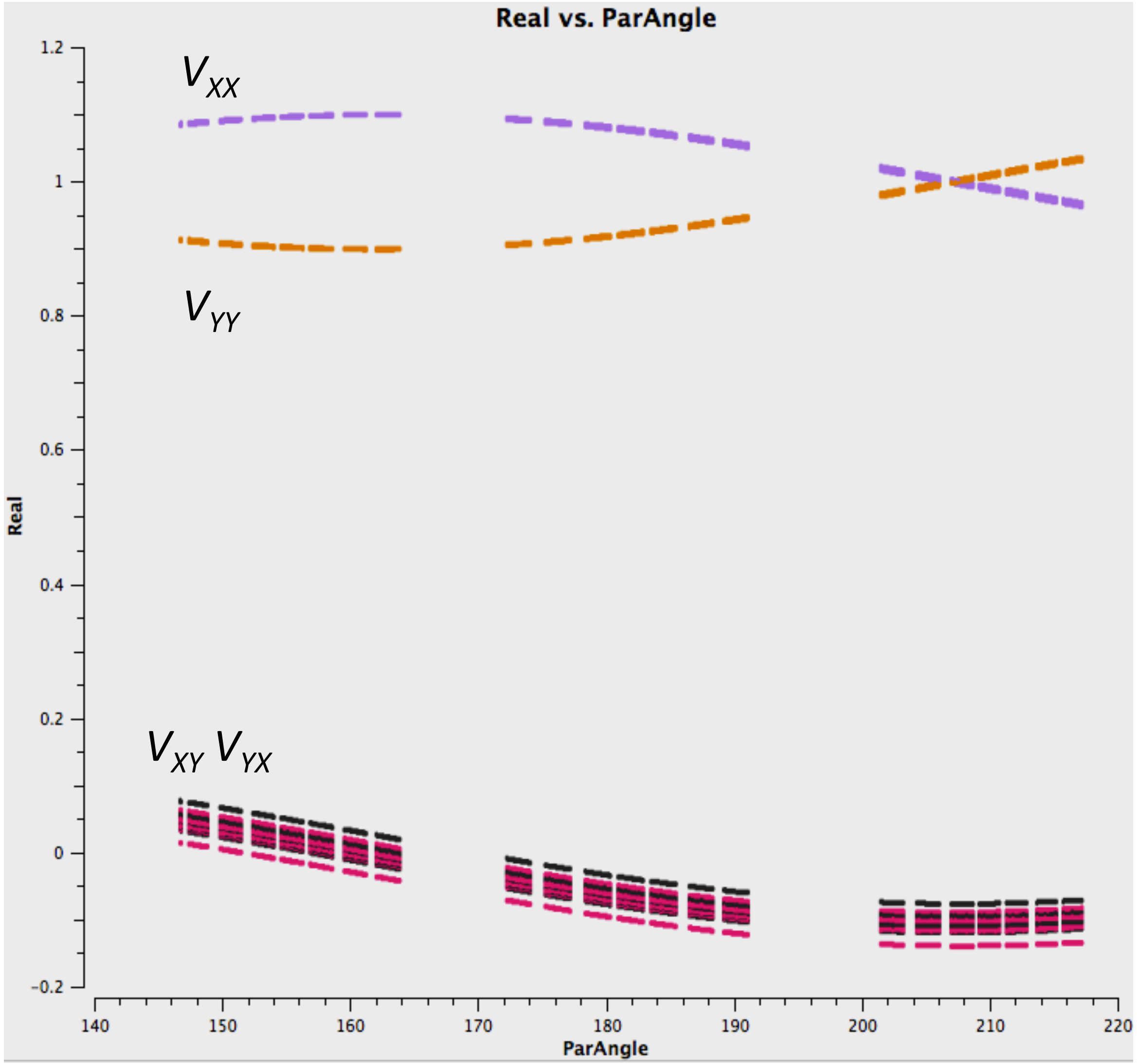
$$V_{XY} = \mathcal{U}_\psi + \mathcal{I}\,(d_{Yj}{}^* + d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}{}^* - d_{Xi})$$

$$V_{YX} = \mathcal{U}_\psi + \mathcal{I}\,(d_{Yi} + d_{Xj}{}^*) + \mathcal{Q}_\psi\,(d_{Yi} - d_{Xj}{}^*)$$

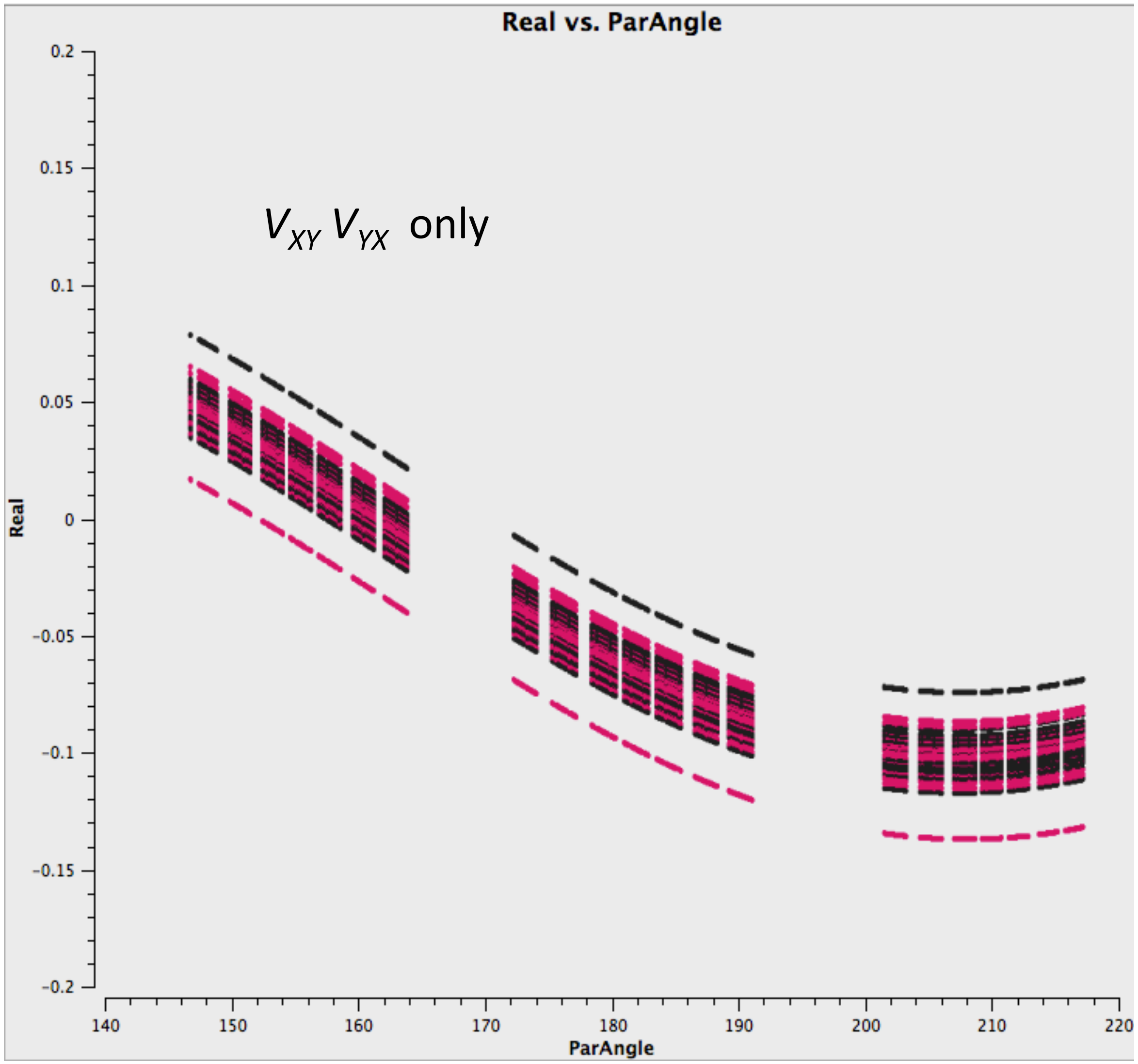
$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Yi} + d_{Yj}{}^*)$$

– (linearized)

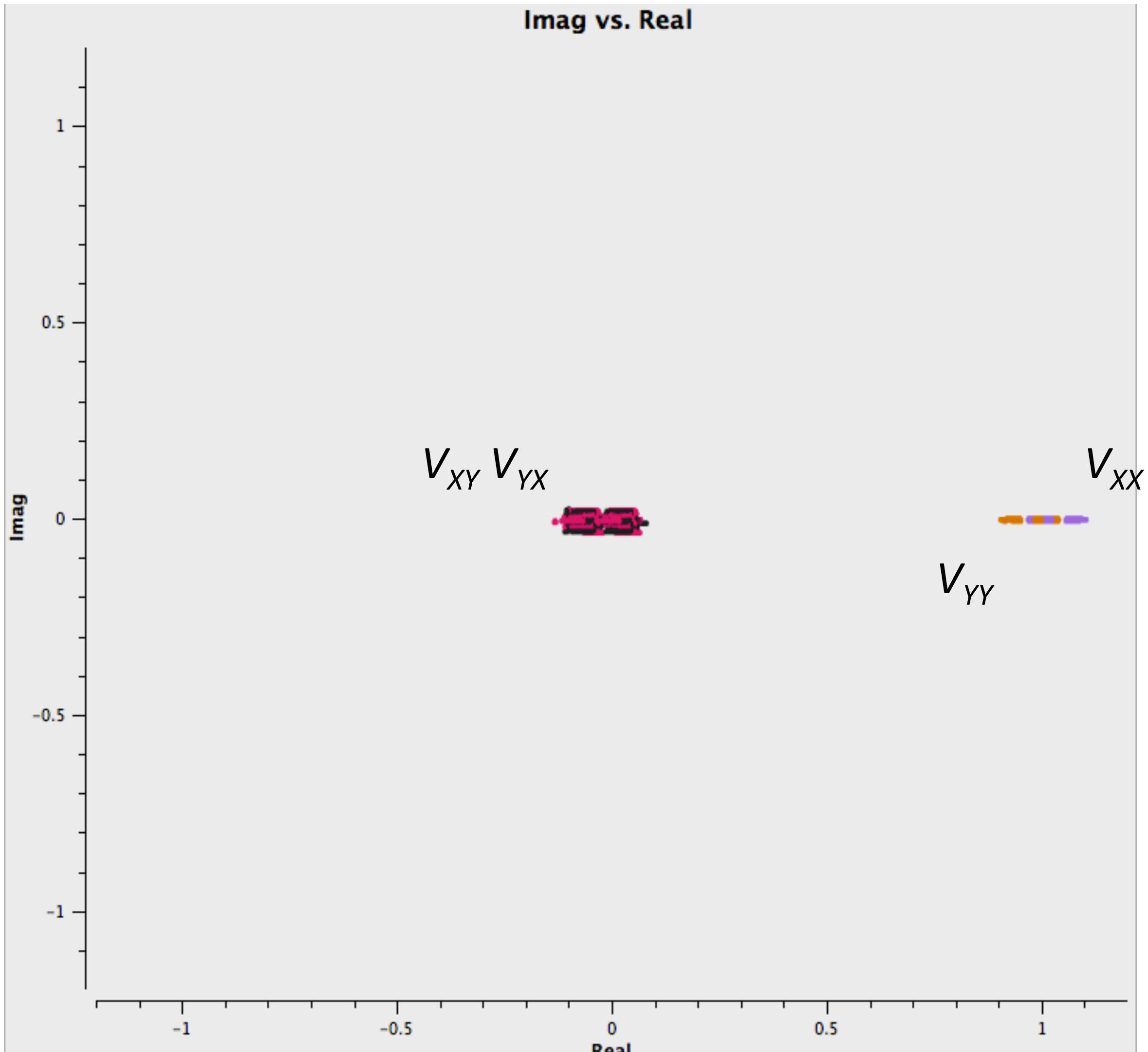
$D\,P\,V^{true}$: Real vs. parang



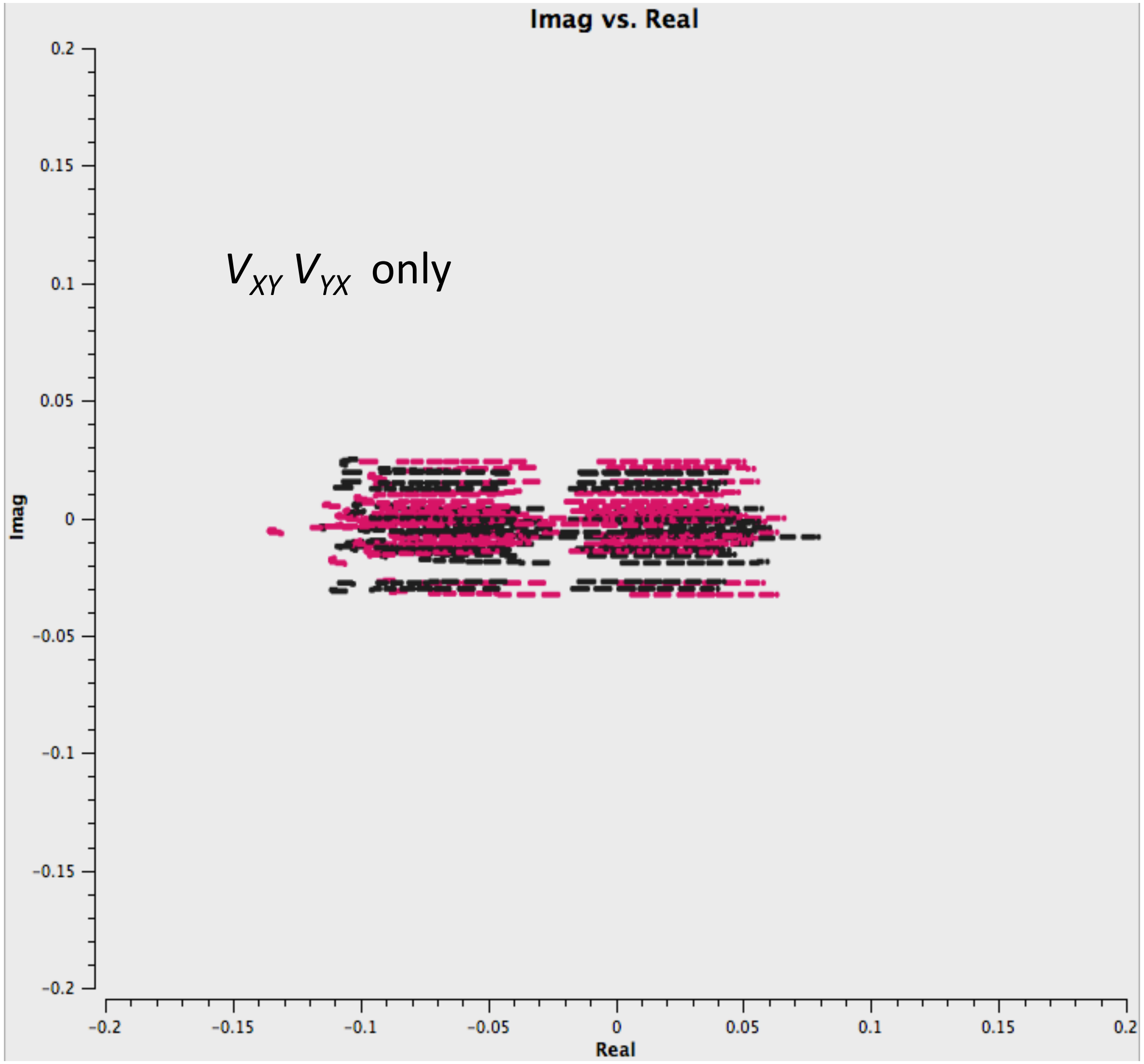
$D\ P\ V^{true}$: Real vs. parang



$D\ P\ V^{true}$: Imag vs. Real



$\boldsymbol{D\,P\,V^{true}}$: Imag vs. Real



Gain: $\boldsymbol{G\ D\ P\ V^{true}}$

$$V_{XX} = g_{Xi}g_{Xj}^* \{(\mathcal{I}+\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Xj}^*+d_{Xi})\}$$

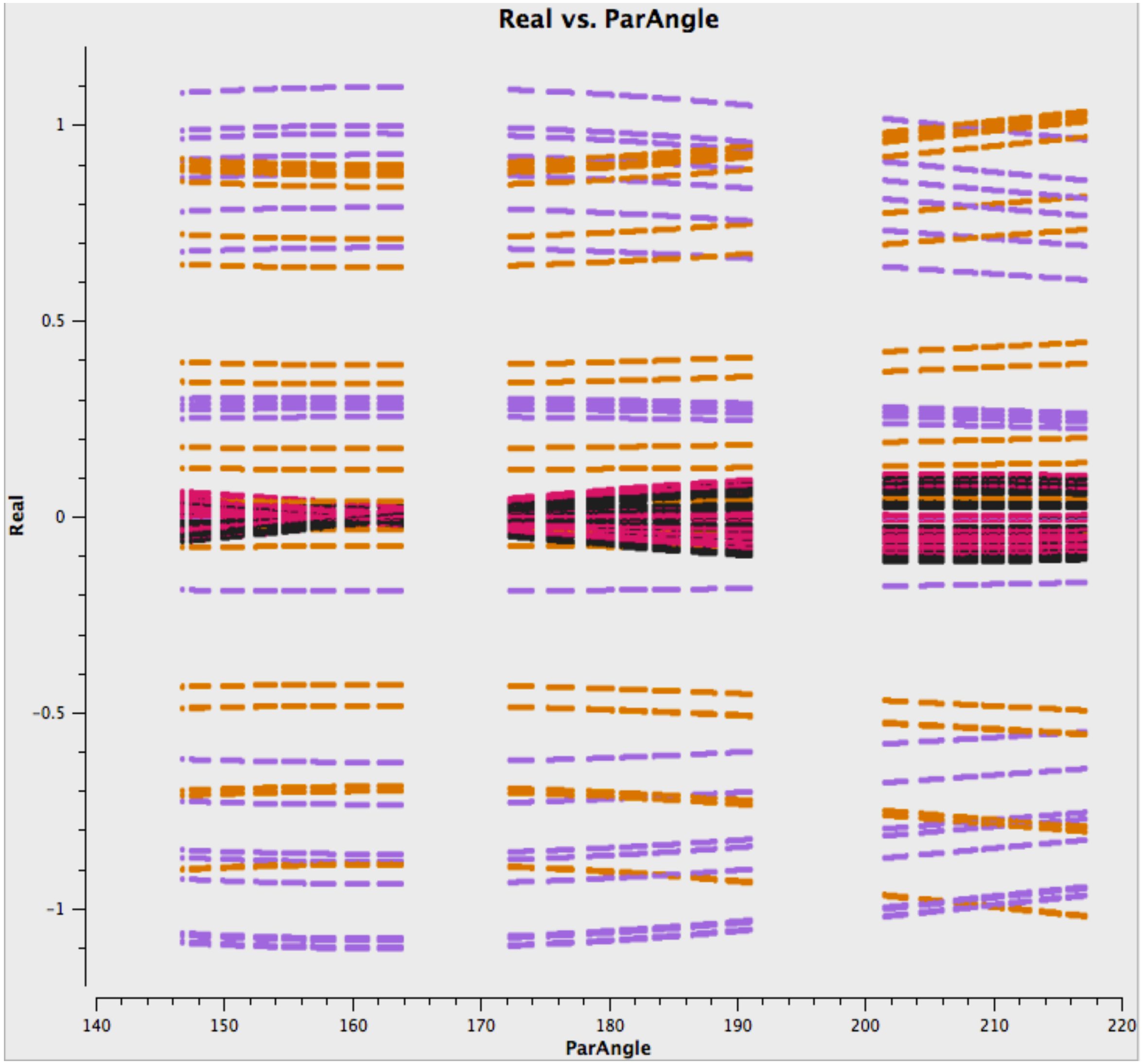
$$V_{XY} = g_{Xi}g_{Yj}^* \{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yj}^*+d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}^*-d_{Xi})\}$$

$$V_{YX} = g_{Yi}g_{Xj}^* \{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yi}+d_{Xj}^*) + \mathcal{Q}_\psi\,(d_{Yi}-d_{Xj}^*)\}$$

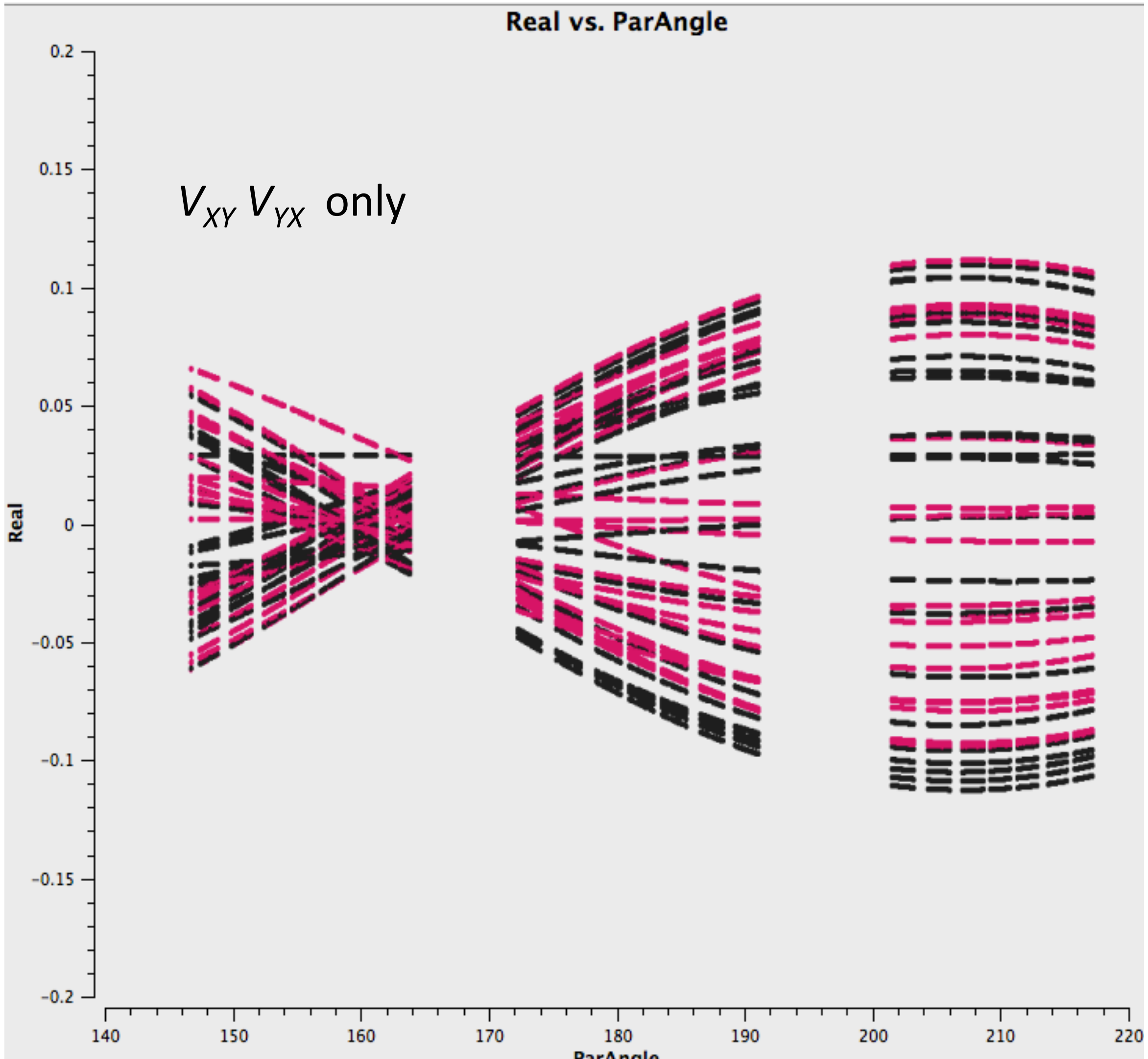
$$V_{YY} = g_{Yi}g_{Yj}^* \{(\mathcal{I}-\mathcal{Q}_\psi) + \mathcal{U}_\psi(d_{Yi}+d_{Yj}^*)\}$$

– For this illustration, ***G*** contains only random, antenna-based phases that are constant in time (i.e., all amplitudes are 1.0)

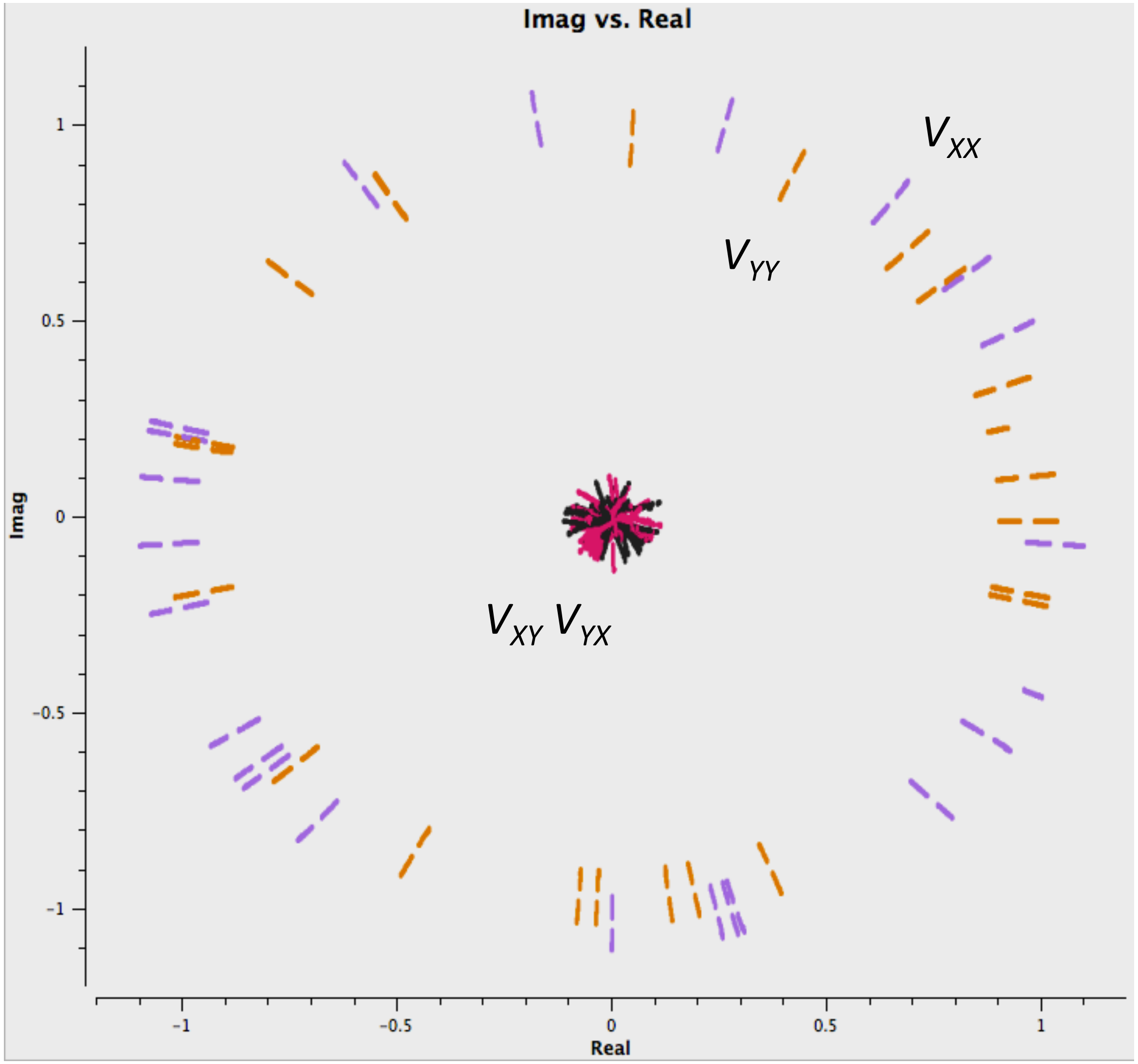
$G\ D\ P\ V^{true}$: Real vs. parang



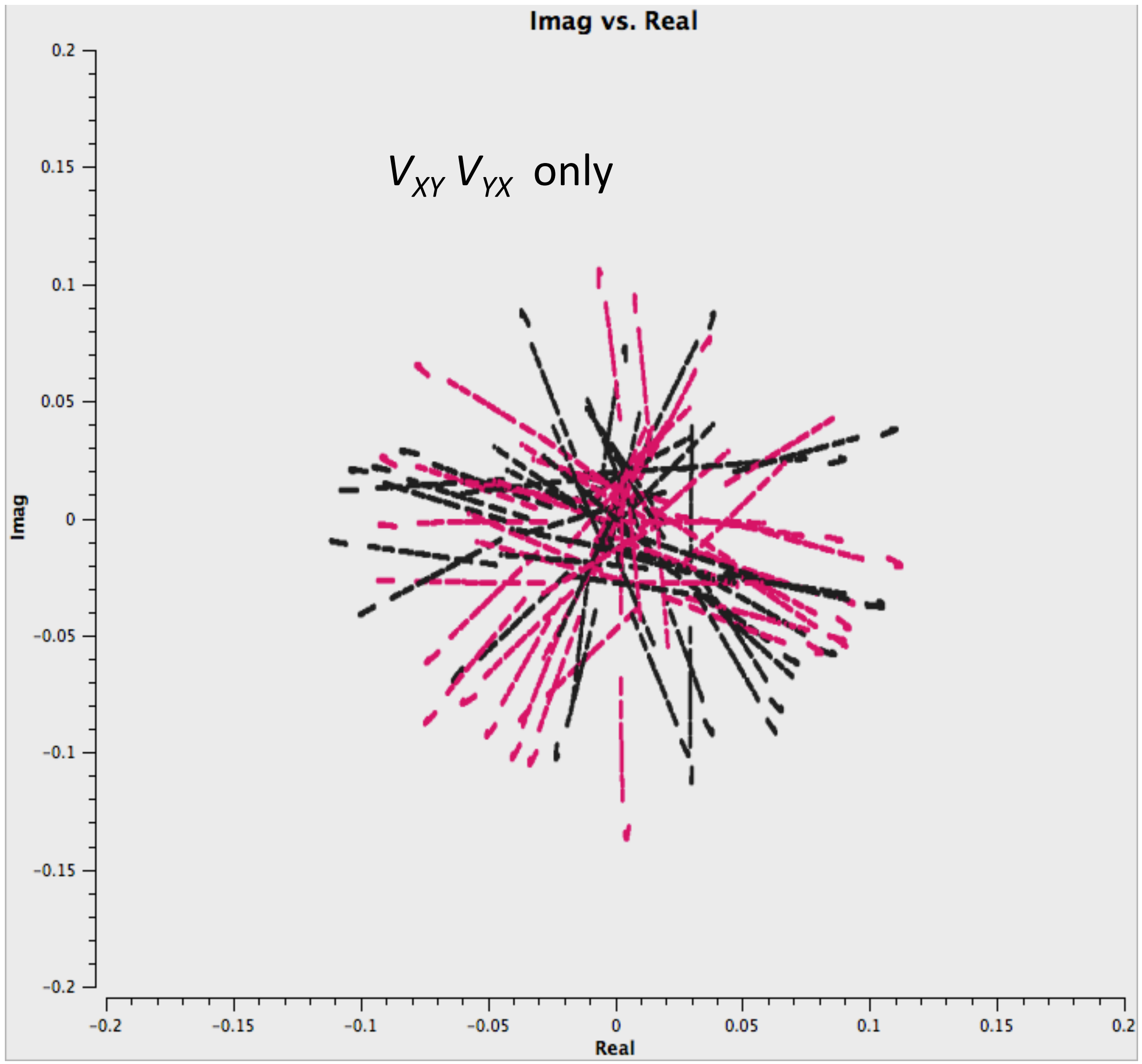
$G\ D\ P\ V^{true}$: Real vs. parang



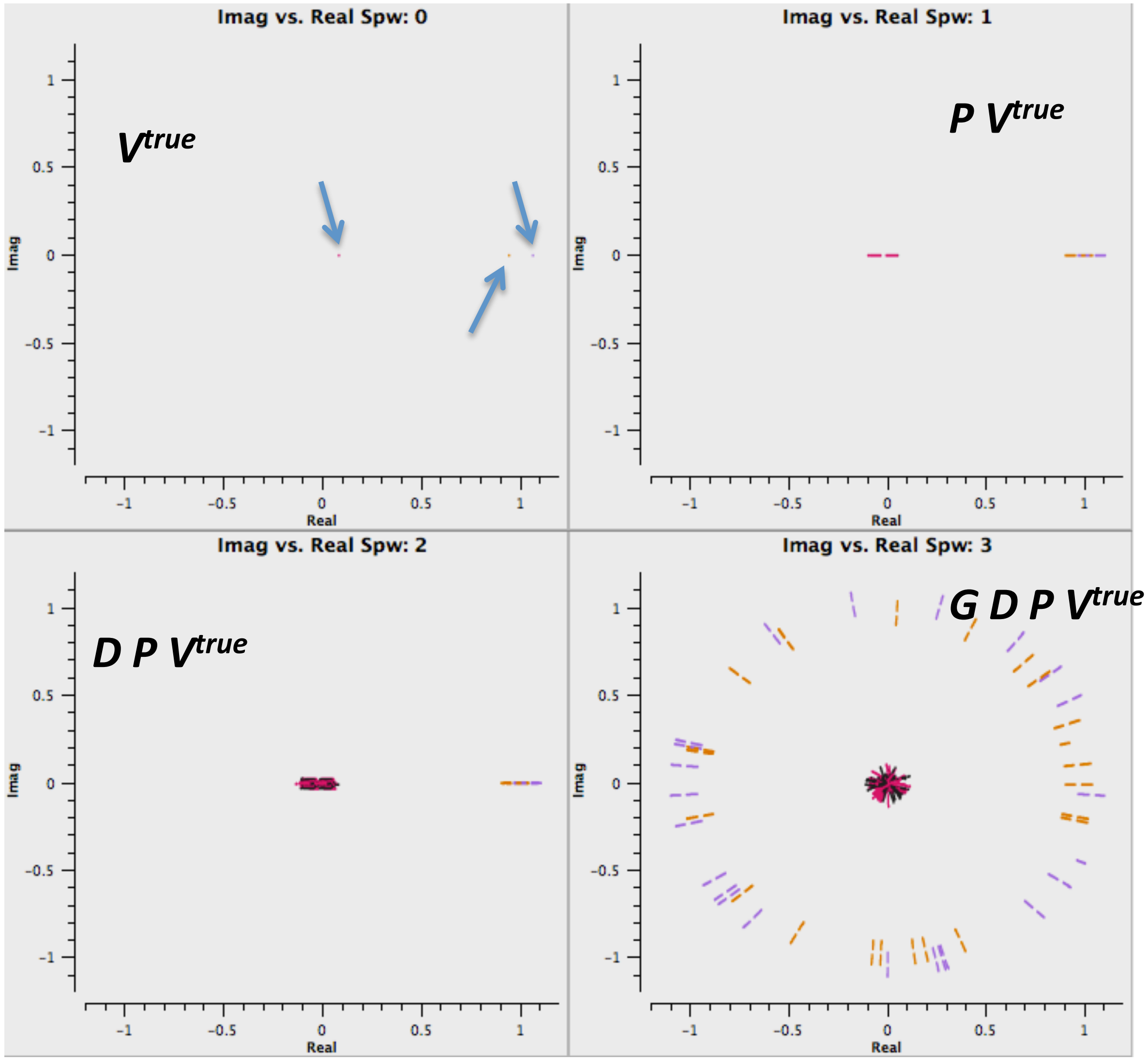
$G\ D\ P\ V^{true}$: Imag vs. Real



$\boldsymbol{G\,D\,P\,V^{true}}$: Imag vs. Real

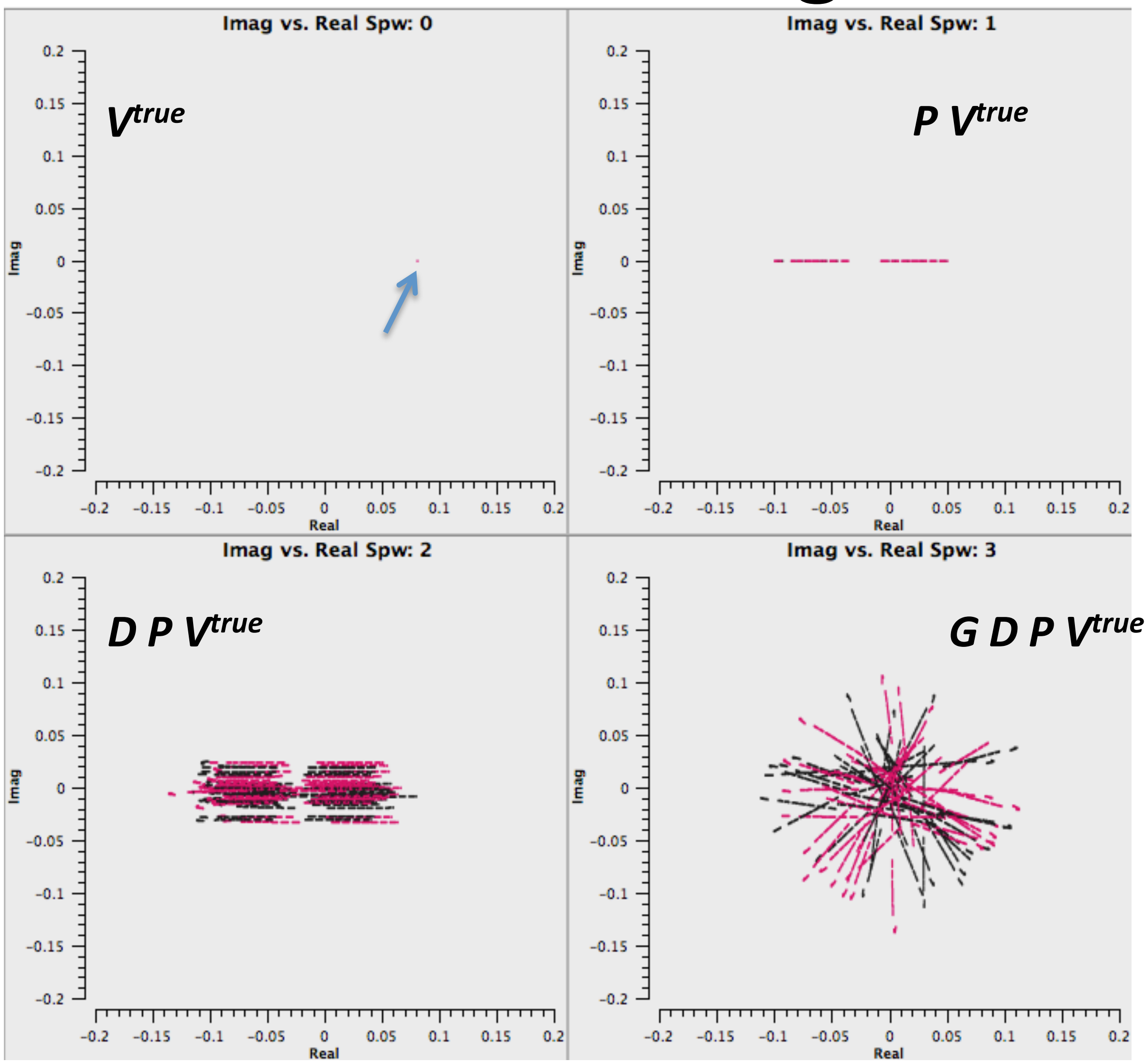


P, D, G Corruption Sequence Real vs. Imag



P, D, G Corruption Sequence Real vs. Imag

$V_{XY}\,V_{YX}$ only



Solving for ***D*** (linear basis)

- Unpolarized source, single scan?
  - Simple, but *d's are* degenerate, to first order:

    $V_{XY} = \mathcal{I}(d_{Yj}^{*}+d_{Xi}) = \mathcal{I}\,[(d_{Yj}-a)^{*}+(d_{Xi}+a^{*})]$

    - for any complex number *a*
  - Therefore, one *d* remains effectively unconstrained
  - Standard convention is to apply a reference antenna that effectively enforces $a = -d_{Xref}^{*}$:

    $$d_{xi} \rightarrow (d_{Xi}+a^{*}) \quad d_{Yi} \rightarrow (d_{Yi}-a) \quad \text{(for all } i\text{)}$$

  - These referenced *d*'s correct the data to some *orthogonal* (to first order) basis that is defined by the refant's true d_X-- but not a *pure* one

Solving for ***D*** (linear basis)

- Polarized source, single scan ($\psi = const$)?
  - Still degenerate in linearized cross-hands:

$$V_{XY} = (\mathcal{I} + \mathcal{Q}_\psi) d_{Yj}{}^* + (\mathcal{I} - \mathcal{Q}_\psi) d_{Xi}$$

$$= (\mathcal{I} + \mathcal{Q}_\psi)(d_{Yj} - b)^* + (\mathcal{I} - \mathcal{Q}_\psi)(d_{Xi} + c^*)$$

    - for any complex numbers *b, c* satisfying

$$b = c(\mathcal{I} + \mathcal{Q}_\psi)/(\mathcal{I} - \mathcal{Q}_\psi)$$

  - Incorrectly calibrates data with different $\mathcal{Q}_\psi$ (other times, sources)

Solving for *D* (linear basis)

- Polarized source w/ parallactic angle coverage
  - Multiple, non-zero $\mathcal{Q}_{\psi}$ breaks degeneracies
  - Depends on time stability of ***D*** (also required for accurate transfer to other sources)
  - Resulting solution accuracy limited by:
    - accuracy of calibrator model's $\mathcal{Q},\mathcal{U}$
    - systematic bias (if any) in assumed feed position angle setting (if $\mathcal{Q},\mathcal{U}$ derived from data)
    - systematic biases (if any) originating in 2nd-order terms

Cross-hand phase spectrum

- An artifact of gain calibration reference antenna (refant)
- We do not measure absolute ***G*** and ***B***
- Instead, we measure $\boldsymbol{G^r}$ and $\boldsymbol{B^r}$, wherein a reference antenna's phase is fixed to zero in both polarizations, yielding relative phases for all other antennas
  - Differences among antennas in each polarization (*separately*) are preserved: no effect on parallel-hand calibration
- The refant's cross-hand bandpass phase remains undetected:

$$\boldsymbol{B\ G = B^r\ G^r\ X^r}$$

- And uncorrected:

$$\boldsymbol{G^{r\ -1}\ {}^rB^{r\ -1}\ B\ G = X^r}$$

$$\boldsymbol{X^r} = \begin{pmatrix} e^{i\rho} & 0 \\ 0 & 1 \end{pmatrix}$$

- $\boldsymbol{X^r}$ is as interesting as any bandpass phase spectrum in the system

Revised factorization

- We therefore rewrite the calibration operator equation:

$$V^{obs} = B\,G\,D\,P\,V^{mod}$$

$$= B^r\,G^r\,X^r\,D\,P\,V^{mod}$$

- It is convenient to move X_r upstream of D:

$$= B^r\,G^r\,D^r\,X^r\,P\,V^{mod}$$

$$(D^r = X^r\,D\,X^{r-1})$$

  - D_r is the instrumental polarization measured in the cross-hand phase frame of the gain & bandpass calibration reference antenna
- In the linear basis, X^r must be determined so cross- and parallel-hands can be combined to extract correct Stokes parameters
- In the circular basis, X^r is just a polarization position angle offset, which can be deferred for later external calibration

Solving for $\boldsymbol{X^r}$ (linear basis)

$$(G^{r\,-1}\, B^{r\,-1}\, V^{obs}) = X_r\, D\, P\, V^{mod}$$

- Consider just the cross-hands:

$$V_{XY} = e^{i\rho}\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yj}{}^* + d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}{}^* - d_{Xi})\}$$
$$V_{YX} = e^{-i\rho}\,\{\mathcal{U}_\psi + \mathcal{I}\,(d_{Yi} + d_{Xj}{}^*) + \mathcal{Q}_\psi\,(d_{Yi} - d_{Xj}{}^*)\}$$

- Average over baselines and correlations:

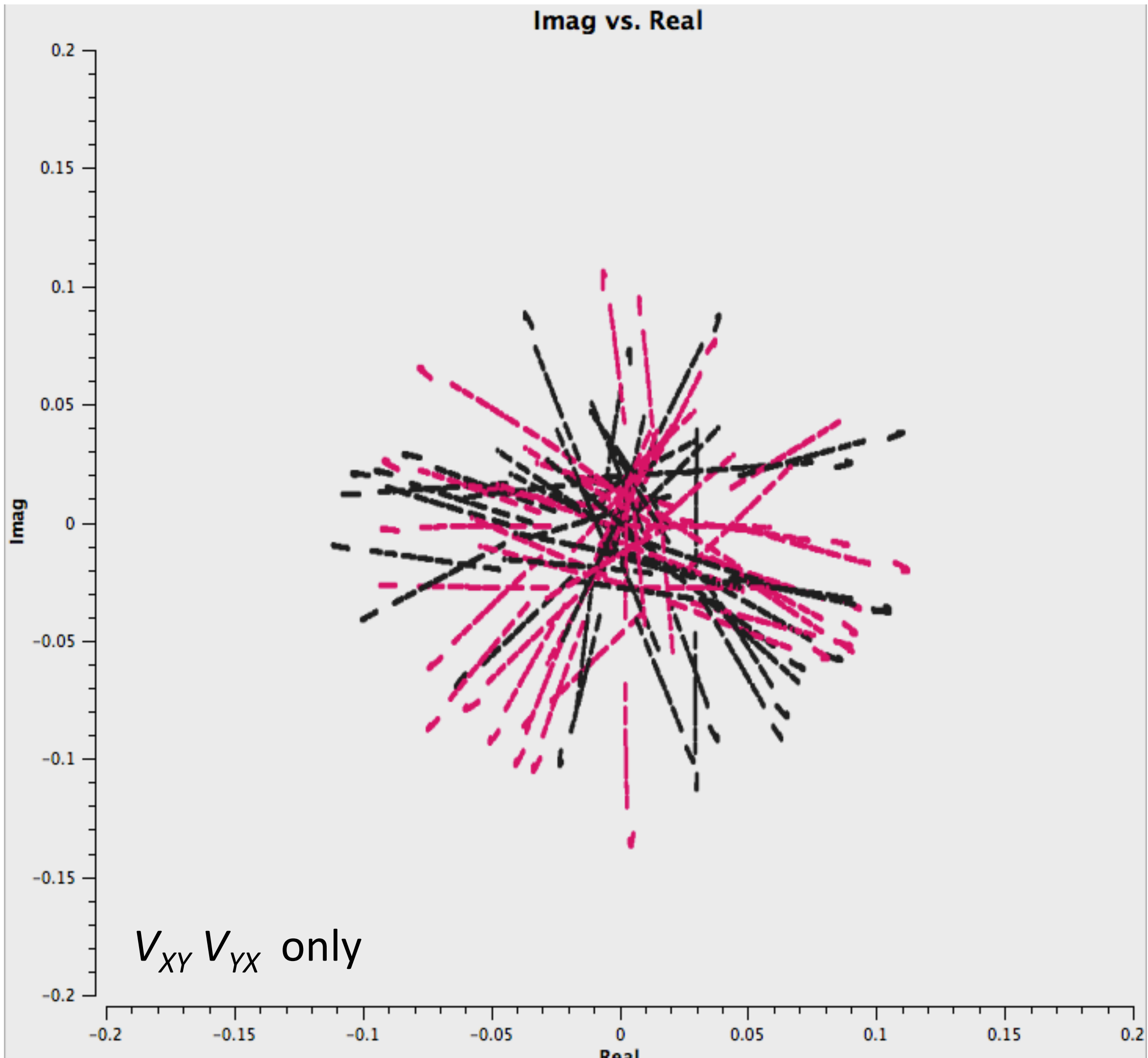
$$(\langle V_{XY}\rangle + \langle V_{YX}{}^*\rangle)/2 = \mathcal{U}_\psi e^{i\rho} + \langle \mathcal{I}(d_{Yj}{}^* + d_{Xi}) + \mathcal{Q}_\psi\,(d_{Yj}{}^* - d_{Xi})\rangle e^{i\rho}$$

$$= (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi)e^{i\rho} + \varepsilon$$

  - ε is small if *d*'s are small and ~random
  - Requires non-zero $\mathcal{Q}$, $\mathcal{U}$ (c.f. desirability for ***D*** solution)
  - Measurements at (at least) 3 distinct ψ sufficient to determine ρ, $\mathcal{Q}$, $\mathcal{U}$, ε
  - Degeneracy: $(\rho, \mathcal{Q}, \mathcal{U}) \rightarrow (\rho+\pi, -\mathcal{Q}, -\mathcal{U})$
    - Resolvable using $\mathcal{Q}$, $\mathcal{U}$ estimate from gain ratio
  - Requires $\boldsymbol{X_r}$ stability: a "good" refant for gain and bandpass (c.f. $|g_X/g_Y|$ stability expectations)
  - ATCA (linears) use a calibration signal to monitor $\boldsymbol{X_r}$
    - refant operation in $\boldsymbol{G^r}$ and $\boldsymbol{B^r}$ solves will then merely enforce the truth)
  - (Circular basis: polarization position angle offset)

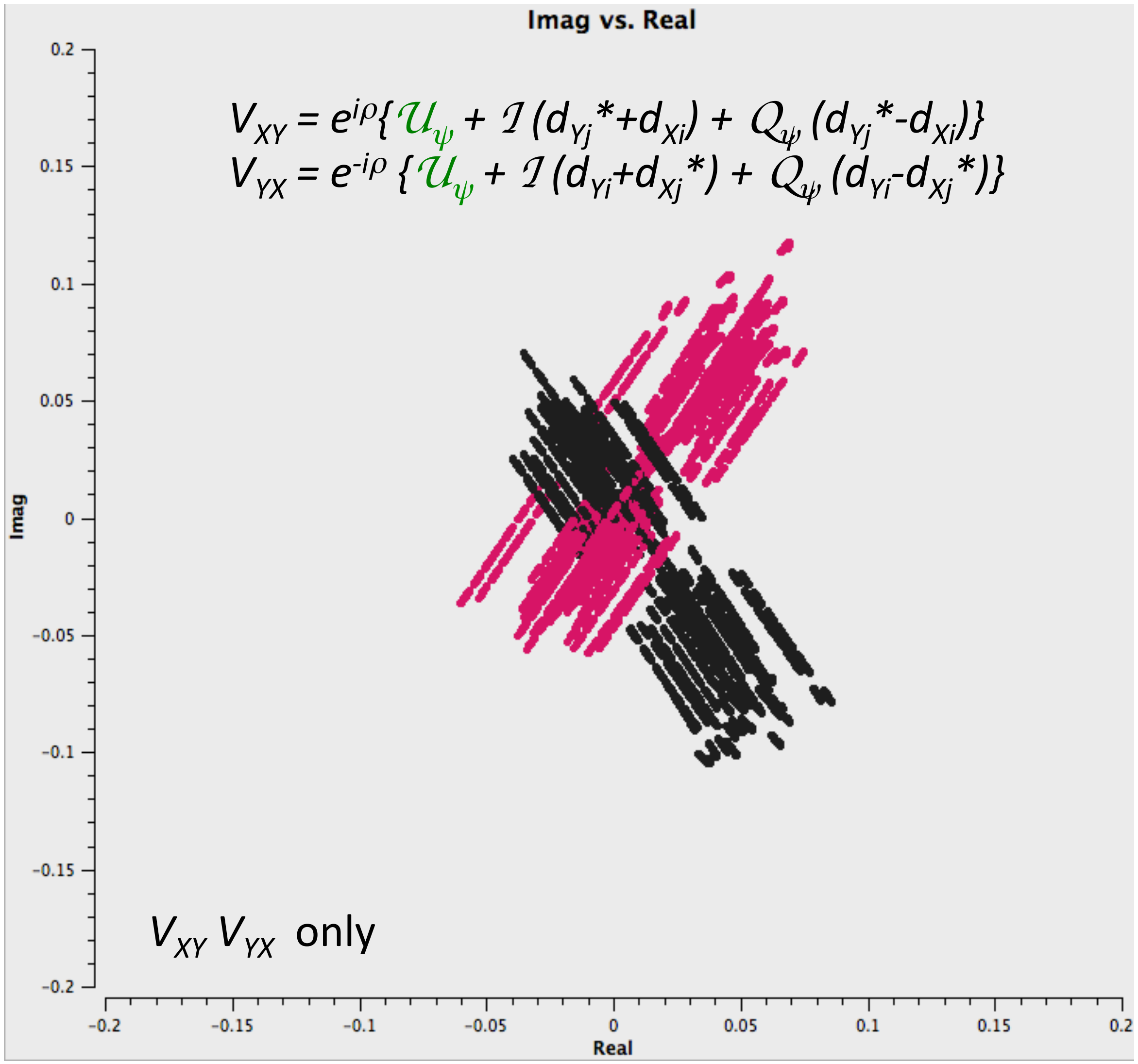
$G\ D\ P\ V^{true}$

Imag vs. Real



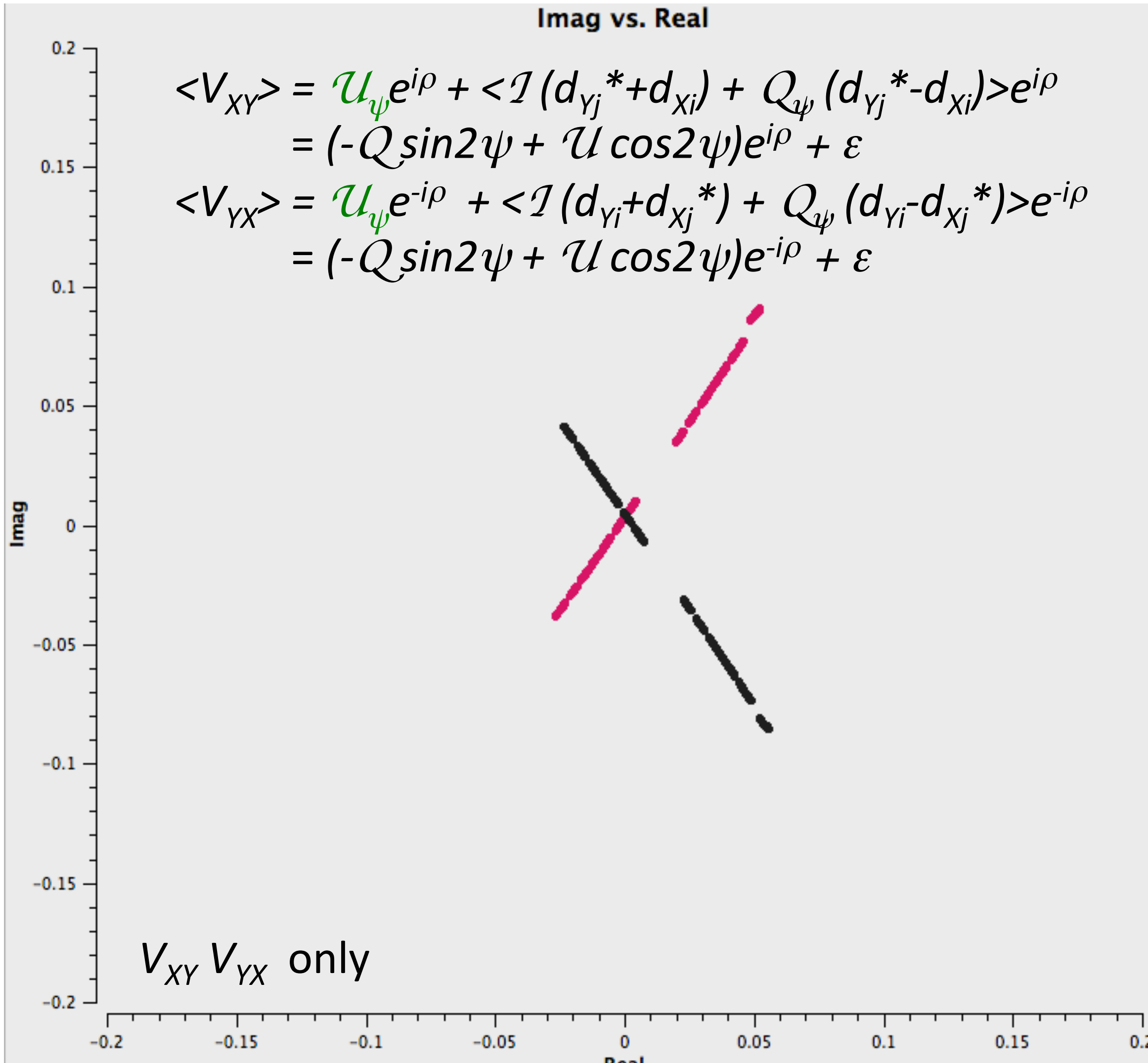
$$(G^{r\,-1}\,G)\,D\,P\,V^{true} = X^r\,D\,P\,V^{true}$$

Imag vs. Real



$\langle X^r\, D\, P\, V^{\mathcal{QU}} \rangle = X^r \langle D \rangle\, P\, V^{\mathcal{QU}}$

Imag vs. Real



Polarization Calibration Bootstrapping

- Calibration Model:

$$V^{obs} = B^r\, G^r\, D^r\, X^r\, P\, V^{mod}$$

- Basic Solve sequence:
  - Normal B^r and G^r (parallel-hands):

$$V^{obs} = \underline{B^r}\, V^{\mathcal{I}}$$

$$(B^{r\,-1}\, V^{obs}) = \underline{G^r}\, V^{\mathcal{I}} \quad \text{(+estimate } \mathcal{QU}\text{)}$$

  - X^r, $\mathcal{Q}$ and $\mathcal{U}$ (cross-hands):

$$(G^{r\,-1}\, B^{r\,-1}\, V^{obs}) = \underline{X^r}\, P\, \underline{V^{\mathcal{QU}}} \quad \text{(+resolve amb)}$$

  - Revise G^r, using IQU (parallel-hands):

$$(B^{r\,-1}\, V^{obs}) = \underline{G^r}\, (P\, V^{\mathcal{IQU}})$$

  - D^r, using IQU (cross-hands):

$$(X^{r\,-1}\, G^{r\,-1}\, B^{r\,-1}\, V^{obs}) = \underline{D^r}\, (X^r\, P\, V^{\mathcal{IQU}})$$

  - (Iteration?)
- Correction:

$$V^{corr} = (P^{-1}\, X^{r\,-1}\, D^{r\,-1}\, G^{r\,-1}\, B^{r\,-1}\, V^{obs})$$